thm_2Ewellorder_2EWF__REC__num (TMWziXQ9YiSd3EJwPy3HnaUT21bznKky6us)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40 A

Definition 4 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap$ (c_2Emin_2E_3D $(2^{2}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ (c_2Emin_2E_3D $(2^{A\_27a}$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0\_3C\_3C \in ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V0\_3C\_3C)) \Rightarrow (\forall V1H \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}). ((\forall V2f \in (A\_27b^{A\_27a}).(\forall V3g \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.((\forall V5z \in A\_27a.((p\ (ap\ (ap\ V0\_3C\_3C\ V5z)\ V4x)) \Rightarrow ((ap\ V2f\ V5z) = (ap\ V3g\ V5z)))) \Rightarrow ((ap\ (ap\ V1H\ V2f)\ V4x) = (ap\ (ap\ V1H\ V3g)\ V4x)))))) \Rightarrow (\exists V6f \in (A\_27b^{A\_27a}).(\forall V7x \in A\_27a.((ap\ V6f\ V7x) = (ap\ (ap\ V1H\ V6f)\ V7x)))))))) \tag{5}$$

Assume the following.

$$(p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C})) \tag{6}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0H \in ((A\_27a^{ty\_2Enum\_2Enum})^{(A\_27a^{ty\_2Enum\_2Enum})}). ((\forall V1f \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V2g \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V3n \in ty\_2Enum\_2Enum.((\forall V4m \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V4m)\ V3n)) \Rightarrow ((ap\ V1f\ V4m) = (ap\ V2g\ V4m)))) \Rightarrow ((ap\ (ap\ V0H\ V1f)\ V3n) = (ap\ (ap\ V0H\ V2g)\ V3n)))))) \Rightarrow (\exists V5f \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V6n \in ty\_2Enum\_2Enum.((ap\ V5f\ V6n) = (ap\ (ap\ V0H\ V5f)\ V6n))))))$$