

thm_2Ewellorder_2EWIN__remove (TMY- poL8ErAD14PfWmo7L5XjkZGtzEYHBR2n)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \tag{3}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_3F))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (4)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ V0x\ V1y))$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 12 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a \times A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (5)$$

Definition 13 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (7)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP \\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \end{aligned} \quad (8)$$

Definition 14 We define $c_2Ewellorder_2Eremove$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 15 We define $c_2Eset_relation_2Erange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 16 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ V1t2)))$

Definition 18 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ V0s\ V1t))$

Definition 19 We define $c_2Eset_relation_2Ereflexive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 20 We define $c_2Eset_relation_2Eantisym$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 21 We define $c_2Eset_relation_2Etransitive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 22 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 23 We define $c_2Eset_relation_2Elinear_order$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 24 We define $c_2Ewellorder_2Ewellfounded$ to be $\lambda A_27a : \iota.\lambda V0R \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 25 We define $c_2Ewellorder_2Ewellorder$ to be $\lambda A_27a : \iota.\lambda V0R \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in A_27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in A_27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b).((ap\ (ap\ (c_2Epair_2E_2C \\ & A.27a\ A.27b)\ (ap\ (c_2Epair_2EFST\ A.27a\ A.27b)\ V0x))\ (ap\ (c_2Epair_2ESND \\ & A.27a\ A.27b)\ V0x)) = V0x)) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}).(\forall V1x \in \\ & A.27a.(\forall V2y \in A.27b.((ap\ (ap\ (c_2Epair_2EUNCURRY\ A.27a \\ & A.27b\ A.27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V1x)\ V2y)) = \\ & (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A.27a\ 2)^{A.27b}).(\forall V1v \in \\ & A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & A.27a\ A.27b)\ V0f))) \Leftrightarrow (\exists V2x \in A.27b.((ap\ (ap\ (c_2Epair_2E_2C \\ & A.27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27a)}). \\ & ((p\ (ap\ (c_2Ewellorder_2Ewellorder\ A.27a)\ V0r)) \Rightarrow ((ap\ (c_2Ewellorder_2Ewellorder_REP \\ & A.27a)\ (ap\ (c_2Ewellorder_2Ewellorder_ABS\ A.27a)\ V0r)) = V0r))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0e \in A.27a.(\forall V1w \in \\ & (ty_2Ewellorder_2Ewellorder\ A.27a).(p\ (ap\ (c_2Ewellorder_2Ewellorder \\ & A.27a)\ (ap\ (c_2Epred_set_2EGSPEC\ (ty_2Epair_2Eprod\ A.27a\ A.27a) \\ & (ty_2Epair_2Eprod\ A.27a\ A.27a))\ (ap\ (c_2Epair_2EUNCURRY\ A.27a \\ & A.27a\ (ty_2Epair_2Eprod\ (ty_2Epair_2Eprod\ A.27a\ A.27a)\ 2))\ (\\ & \lambda V2x \in A.27a.(\lambda V3y \in A.27a.(ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Epair_2Eprod \\ & A.27a\ A.27a)\ 2)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V2x)\ V3y)) \\ & (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ c_2Ebool_2E_7E\ (ap\ (ap\ (c_2Emin_2E_3D \\ & A.27a)\ V2x)\ V0e)))\ (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ c_2Ebool_2E_7E \\ & (ap\ (ap\ (c_2Emin_2E_3D\ A.27a)\ V3y)\ V0e)))\ (ap\ (ap\ (c_2Ebool_2EIN \\ & (ty_2Epair_2Eprod\ A.27a\ A.27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a \\ & A.27a)\ V2x)\ V3y))\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A.27a) \\ & V1w)))))))))) \end{aligned} \quad (21)$$

Theorem 1

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (\forall V1y \in \\ & \quad A_{27a}. (\forall V2e \in A_{27a}. (\forall V3w \in (\text{ty_2Ewellorder_2Ewellorder} \\ & \quad A_{27a}). (p \text{ (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A_{27a} A_{27a})) \\ & \quad (\text{ap (ap (c_2Epair_2E_2C A_{27a} A_{27a}) V0x) V1y)) (ap (c_2Eset_relation_2Estrict} \\ & \quad A_{27a}) (\text{ap (c_2Ewellorder_2Ewellorder_REP A_{27a}) (ap (ap (c_2Ewellorder_2Eremove} \\ & \quad A_{27a}) V2e) V3w)))))) \Leftrightarrow ((\neg(V0x = V2e)) \wedge (\neg(V1y = V2e)) \wedge (p \text{ (ap (ap} \\ & \quad (\text{c_2Ebool_2EIN (ty_2Epair_2Eprod A_{27a} A_{27a})) (ap (ap (c_2Epair_2E_2C} \\ & \quad A_{27a} A_{27a}) V0x) V1y)) (ap (c_2Eset_relation_2Estrict A_{27a}) \\ & \quad (\text{ap (c_2Ewellorder_2Ewellorder_REP A_{27a}) V3w)))))))))) \end{aligned}$$