

thm_2Ewellorder_2EWIN__trichotomy (TMRnrM- MDsHLU5Rz81WyW3gzRxA5jdzzZMuY)

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Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2)))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a))))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_2F))$

Definition 9 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 25 We define $c_Ewellorder_2EelsOf$ to be $\lambda A.27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{13}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \tag{15}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \tag{16}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \tag{18}$$

Assume the following.

$$2.((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{.27})) \wedge ((p \ V1x_{.27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{.27})))))) \Rightarrow ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{.27}) \Rightarrow (p \ V3y_{.27})))))) \Rightarrow \quad (19)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (ap \ V0P \ V1a)))))) \quad (20)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap \ (ap \ (c_{.2Epair_2E_2C} \ A_{.27a} \ A_{.27b}) \ V0x) \ V1y) = (ap \ (ap \ (c_{.2Epair_2E_2C} \ A_{.27a} \ A_{.27b}) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (21)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap \ (ap \ (c_{.2Epair_2E_2C} \ A_{.27a} \ A_{.27b}) \ V0x) \ V1y) = (ap \ (ap \ (c_{.2Epair_2E_2C} \ A_{.27a} \ A_{.27b}) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (22)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0x \in (ty_{.2Epair_2Eprod} \ A_{.27a} \ A_{.27b}).((ap \ (ap \ (c_{.2Epair_2E_2C} \ A_{.27a} \ A_{.27b}) \ (ap \ (c_{.2Epair_2EFST} \ A_{.27a} \ A_{.27b}) \ V0x)) \ (ap \ (c_{.2Epair_2ESND} \ A_{.27a} \ A_{.27b}) \ V0x)) = V0x)) \quad (23)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow \forall A_{.27c}.nonempty \ A_{.27c} \Rightarrow (\forall V0f \in ((A_{.27c}^{A_{.27b}})^{A_{.27a}}).(\forall V1x \in A_{.27a}.(\forall V2y \in A_{.27b}.((ap \ (ap \ (c_{.2Epair_2EUNCURRY} \ A_{.27a} \ A_{.27b} \ A_{.27c}) \ V0f) \ (ap \ (ap \ (c_{.2Epair_2E_2C} \ A_{.27a} \ A_{.27b}) \ V1x) \ V2y))) = (ap \ (ap \ V0f \ V1x) \ V2y)))))) \quad (24)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty \ A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2Epair_2Eprod} \ A_{.27a} \ 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p \ (ap \ (ap \ (c_{.2Ebool_2EIN} \ A_{.27a}) \ V1v) \ (ap \ (c_{.2Epred_set_2EGSPEC} \ A_{.27a} \ A_{.27b}) \ V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap \ (ap \ (c_{.2Epair_2E_2C} \ A_{.27a} \ 2) \ V1v) \ c_{.2Ebool_2ET}) = (ap \ V0f \ V2x)))))) \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred_set_2EUNION\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (26) \\ & (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c.2Ebool_2EIN \\ & A.27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(p\ (ap\ (ap\ (c.2Epred_set_2ESUBSET\ A.27a)\ V0s)\ (ap\ (\\ & ap\ (c.2Epred_set_2EUNION\ A.27a)\ V0s)\ V1t)))) \wedge (\forall V2s \in (27) \\ & (2^{A.27a}).(\forall V3t \in (2^{A.27a}).(p\ (ap\ (ap\ (c.2Epred_set_2ESUBSET \\ & A.27a)\ V2s)\ (ap\ (ap\ (c.2Epred_set_2EUNION\ A.27a)\ V3t)\ V2s)))))) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (31) \end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (33) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (34) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q)) \Rightarrow (p \ V0p))) \tag{38}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q)) \Rightarrow \neg(p \ V1q)))) \tag{39}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \vee (p \ V1q)) \Rightarrow \neg(p \ V0p)))) \tag{40}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \vee (p \ V1q)) \Rightarrow \neg(p \ V1q)))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p \ V0p)) \Rightarrow (p \ V0p))) \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty \ A_27a \Rightarrow ((\forall V0a \in (ty_2Ewellorder_2Ewellorder \\
& A_27a). ((ap \ (c_2Ewellorder_2Ewellorder_ABS \ A_27a) \ (ap \ (c_2Ewellorder_2Ewellorder_REP \\
& A_27a) \ V0a)) = V0a)) \wedge (\forall V1r \in (2^{(ty_2Epair_2Eprod \ A_27a \ A_27a)}). \\
& ((p \ (ap \ (c_2Ewellorder_2Ewellorder \ A_27a) \ V1r)) \Leftrightarrow ((ap \ (c_2Ewellorder_2Ewellorder_REP \\
& A_27a) \ (ap \ (c_2Ewellorder_2Ewellorder_ABS \ A_27a) \ V1r)) = V1r))))
\end{aligned} \tag{43}$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\ A_27a).(\forall V1x \in A_27a.(\forall V2y \in A_27a.(((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V1x)\ (ap\ (c_2Ewellorder_2EelsOf\ A_27a)\ V0w)))) \wedge (p\ (ap\ (ap \\ (c_2Ebool_2EIN\ A_27a)\ V2y)\ (ap\ (c_2Ewellorder_2EelsOf\ A_27a) \\ V0w)))))) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V1x)\ V2y))\ (ap\ (c_2Eset_relation_2Estrict \\ A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A_27a)\ V0w)))))) \vee (\\ (V1x = V2y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V2y)\ V1x))\ (ap\ (c_2Eset_relation_2Estrict \\ A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A_27a)\ V0w)))))))))) \end{aligned}$$