

thm_2Ewellorder_2EWIN_wZERO (TM- RAe8P2VNH4GrQijWCAAYLYFC4te7MvYsX)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V 0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V 0t \in 2.V 0t))$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. nonempty A 0 \Rightarrow \forall A 1. nonempty A 1 \Rightarrow nonempty (ty_2Epair_2Eprod A 0 A 1) \tag{1}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \tag{3}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. if (\exists x \in A. p (ap P x))$ then (the $(\lambda x. x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 6 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V 0P \in (2^{A_27a}). (ap V 0P (ap (c_2Emin_2E_40 A_27a P)))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V 0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V 0t) c_2Ebool_2E_3F))$

Definition 9 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V 0x \in A_27a. (\lambda V 1f \in (2^{A_27a}). (ap V 1f V 0x)))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (4)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Definition 12 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 13 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ewellorder_2Ewellorder A0) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP \\ A_27a \in ((2^{(ty_2Epair_2Eprod A_27a A_27a)})^{(ty_2Ewellorder_2Ewellorder A_27a)}) \end{aligned} \quad (7)$$

Definition 14 We define $c_2Eset_relation_2Erange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A$

Definition 15 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 17 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 18 We define $c_2Eset_relation_2Ereflexive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 19 We define $c_2Eset_relation_2Eantisym$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 20 We define $c_2Eset_relation_2Etransitive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 21 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 22 We define $c_2Eset_relation_2Elinear_order$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 23 We define $c_2Ewellorder_2Ewellfounded$ to be $\lambda A_27a : \iota.\lambda V0R \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 24 We define $c_2Ewellorder_2Ewellorder$ to be $\lambda A_27a : \iota.\lambda V0R \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 25 We define $c_Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}})) \end{aligned} \quad (8)$$

Definition 26 We define $c_2Ewellorder_2EwZERO$ to be $\lambda A_27a : \iota. (ap\ (c_2Ewellorder_2Ewellorder_ABS$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in \\ & A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p\ V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ A_27b)\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0x))\ (ap\ (c_2Epair_2ESND \\ & \quad A_27a\ A_27b)\ V0x)) = V0x)) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\ & \quad A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ & \quad A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\ & \quad (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (c_2Ewellorder_2Ewellorder\ A_27a)\ (c_2Epred_set_2EEMPTY\ (ty_2Epair_2Eprod\ A_27a\ A_27a)))) \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ & \quad ((p\ (ap\ (c_2Ewellorder_2Ewellorder\ A_27a)\ V0r)) \Rightarrow ((ap\ (c_2Ewellorder_2Ewellorder_REP \\ & \quad A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ V0r)) = V0r))) \end{aligned} \quad (24)$$

Theorem 1

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (\forall V1y \in A_{27a}. ((p (ap (ap (c_Ebool_2EIN (ty_2Epair_2Eprod A_{27a} A_{27a})) (ap (ap (c_Epair_2E_2C A_{27a} A_{27a}) V0x) V1y)) (ap (c_Eset_relation_2Estrict A_{27a}) (ap (c_Ewellorder_2Ewellorder_REP A_{27a}) (c_Ewellorder_2EwZERO A_{27a})))))) \Leftrightarrow False)))$$