

thm_2Ewellorder_2EdestWO_mkWO (TMNyPunuPBwafd4ZGTySvtxSnjm4L8Vviso)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) (\lambda V2z \in 2.V2z))$

Definition 7 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) (\lambda V1x \in A_27a.V1x))))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A_27a 2)^{A-27b}}) \tag{3}$$

Definition 10 We define $c_Eset_relation_Erange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 11 We define $c_Eset_relation_Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V0t1))))$

Definition 13 We define $c_Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_Epred_set_2EUNION\ V0s\ V1t))$

Definition 14 We define $c_Eset_relation_Ereflexive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 15 We define $c_Eset_relation_Eantisym$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 16 We define $c_Eset_relation_Etransitive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 17 We define $c_Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_Epred_set_2ESUBSET\ V0s\ V1t))$

Definition 18 We define $c_Eset_relation_Elinear_order$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 19 We define c_Ebool_2EF to be $(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 20 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2E_7E))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (5)$$

Definition 21 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Definition 22 We define $c_Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 23 We define $c_2Ewellorder_2Ewellfounded$ to be $\lambda A_27a : \iota.\lambda V0R \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 24 We define $c_2Ewellorder_2Ewellorder$ to be $\lambda A_27a : \iota.\lambda V0R \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP \\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \end{aligned} \quad (7)$$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}}) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in (ty_2Ewellorder_2Ewellorder \\ & A_27a).((ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\ & A_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ & ((p\ (ap\ (c_2Ewellorder_2Ewellorder\ A_27a)\ V1r)) \Leftrightarrow ((ap\ (c_2Ewellorder_2Ewellorder_REP \\ & A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ V1r)) = V1r)))) \end{aligned} \quad (9)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\ & ((p\ (ap\ (c_2Ewellorder_2Ewellorder\ A_27a)\ V0r)) \Rightarrow ((ap\ (c_2Ewellorder_2Ewellorder_REP \\ & A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ V0r)) = V0r))) \end{aligned}$$