

thm_2Ewellorder_2Eorderiso__wZERO (TM- PwN4SZMd8d9EToWRyr4eHuBmiY8n9atv5)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 9 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2E_2C$

Definition 10 We define `c_2Ebool_2E_2IN` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 11 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(3)

Definition 13 We define $c_2Eset_relation_2Erange$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 14 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 15 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Emin_2E_40$

Definition 16 We define $c_2Eset_relation_2Ereflexive$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 17 We define $c_2Eset_relation_2Eantisym$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 18 We define $c_2Eset_relation_2Etransitive$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 19 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Emin_2E_40$

Definition 20 We define $c_2Eset_relation_2Elinear_order$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$$
(4)

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$$
(5)

Definition 21 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Definition 22 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 23 We define $c_2Ewellorder_2Ewellfounded$ to be $\lambda A_27a : \iota. \lambda V0R \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 24 We define $c_2Ewellorder_2Ewellorder$ to be $\lambda A_27a : \iota. \lambda V0R \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 25 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27a)$

Definition 26 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27a)$

Definition 27 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27a)$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP \\ & A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})(ty_2Ewellorder_2Ewellorder\ A_27a)) \end{aligned} \quad (7)$$

Definition 28 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 29 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (8)$$

Definition 30 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 31 We define $c_2Ewellorder_2EwZERO$ to be $\lambda A_27a : \iota. (ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a))$.

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in \\ & A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (23)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}).((\forall V1p \in \\ & (ty_2Epair_2Eprod\ A_27a\ A_27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow (\forall V2p_1 \in \\ & A_27a.(\forall V3p_2 \in A_27b.(p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ A_27b)\ V2p_1)\ V3p_2))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\neg(p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).(\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t))))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).((\forall V1s \in (2^{A_27b}).((p\ (ap\ (ap \\ & (ap\ (c_2Epred_set_2EBIJ\ A_27a\ A_27b)\ V0f)\ (c_2Epred_set_2EEMPTY \\ & A_27a)\ V1s)) \Leftrightarrow (V1s = (c_2Epred_set_2EEMPTY\ A_27b)))))) \wedge (\forall V2s \in \\ & (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ A_27a\ A_27b)\ V0f) \\ & V2s)\ (c_2Epred_set_2EEMPTY\ A_27b))) \Leftrightarrow (V2s = (c_2Epred_set_2EEMPTY \\ & A_27a))))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a.(\forall V1r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Eset_relation_2Edomain \\ & A_27a\ A_27b)\ V1r))) \Leftrightarrow (\exists V2y \in A_27b.(p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\ & A_27b)\ V0x)\ V2y)\ V1r))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0y \in A_27a. (\forall V1r \in (2^{(ty_2Epair_2Eprod\ A_27b\ A_27a)}). \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0y)\ (ap\ (c_2Eset_relation_2Erange \\
& \quad \quad A_27a\ A_27b)\ V1r))) \Leftrightarrow (\exists V2x \in A_27b. (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad \quad (ty_2Epair_2Eprod\ A_27b\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b \\
& \quad \quad \quad A_27a)\ V2x)\ V0y))\ V1r))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (c_2Ewellorder_2Ewellorder\ A_27a)\ (c_2Epred_set_2EEMPTY\ (ty_2Epair_2Eprod\ A_27a\ A_27a)))) \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in (ty_2Ewellorder_2Ewellorder \\
& \quad A_27a). ((ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\
& \quad \quad A_27a)\ V0a)) = V0a) \wedge (\forall V1r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\
& \quad ((p\ (ap\ (c_2Ewellorder_2Ewellorder\ A_27a)\ V1r)) \Leftrightarrow ((ap\ (c_2Ewellorder_2Ewellorder_REP \\
& \quad \quad A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ V1r)) = V1r))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\
& \quad ((p\ (ap\ (c_2Ewellorder_2Ewellorder\ A_27a)\ V0r)) \Rightarrow ((ap\ (c_2Ewellorder_2Ewellorder_REP \\
& \quad \quad A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ V0r)) = V0r)))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\
& \quad A_27a). (\exists V1s \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). ((p \\
& \quad (ap\ (c_2Ewellorder_2Ewellorder\ A_27a)\ V1s) \wedge (V0w = (ap\ (c_2Ewellorder_2Ewellorder_ABS \\
& \quad \quad \quad A_27a)\ V1s))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A.27a). (\forall V1w2 \in \\
& \quad (ty_2Ewellorder_2Ewellorder\ A.27b). ((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\
& \quad A.27a\ A.27b)\ V0w1)\ V1w2)) \Leftrightarrow (\exists V2f \in (A.27b^{A.27a}). ((p\ (ap\ (\\
& \quad ap\ (ap\ (c_2Epred_set_2EBIJ\ A.27a\ A.27b)\ V2f)\ (ap\ (c_2Ewellorder_2EelsOf \\
& \quad A.27a)\ V0w1))\ (ap\ (c_2Ewellorder_2EelsOf\ A.27b)\ V1w2))) \wedge (\forall V3x \in \\
& \quad A.27a. (\forall V4y \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\
& \quad A.27a\ A.27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V3x)\ V4y))\ (ap \\
& \quad (c_2Eset_relation_2Estrict\ A.27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\
& \quad A.27a)\ V0w1)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27b \\
& \quad A.27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27b\ A.27b)\ (ap\ V2f\ V3x))\ (ap\ V2f \\
& \quad V4y)))\ (ap\ (c_2Eset_relation_2Estrict\ A.27b)\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\
& \quad A.27b)\ V1w2)))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c_2Ewellorder_2EelsOf\ A.27a) \\
(c_2Ewellorder_2EwZERO\ A.27a)) = (c_2Epred_set_2EEMPTY\ A.27a)) \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& \quad A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27a\ A.27a)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V0x)\ V1y))\ (ap\ (c_2Eset_relation_2Estrict \\
& \quad A.27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A.27a)\ (c_2Ewellorder_2EwZERO \\
& \quad A.27a)))))) \Leftrightarrow False))
\end{aligned} \tag{38}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0w \in (ty_2Ewellorder_2Ewellorder\ A.27b). ((p\ (ap\ (ap\ (\\
& \quad c_2Ewellorder_2Eorderiso\ A.27a\ A.27b)\ (c_2Ewellorder_2EwZERO \\
& \quad A.27a))\ V0w)) \Leftrightarrow (V0w = (c_2Ewellorder_2EwZERO\ A.27b)))
\end{aligned}$$