thm_2Ewellorder_2Eorderlt__TRANS (TMco53dLMMkxFLRUCRgkPvZhH1r6STYym8a)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{2}))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2EF

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{2}$$

Let $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{3}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \tag{4}$$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{5}$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{6}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{7}$$

Definition 12 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 13 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 15 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A}$

Definition 16 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

Definition 17 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 18 *We define* c_2Ewellorder_2EelsOf *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A$

Definition 19 *We define* c_2Ewellorder_2Eorderiso *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2$

Definition 20 *We define* c_2Ewellorder_2Eiseg *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_2$

Definition 21 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Let $c\_2Ewellorder\_2Ewellorder\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{8}$$

Definition 22 *We define* c_2Ewellorder_2Ewobound *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1w \in (ty\_2Ewellord$

Definition 23 *We define* c_2Ewellorder_2Eorderlt *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2E$

Definition 24 *We define* c_2Epred__set_2ESURJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in ($

Definition 25 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 26 *We define* c_2Epred__set_2EBIJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{13}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{20}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{21}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{23}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27a\ A\_27b)\ V0f)\ V1s)\ V2t)) \Leftrightarrow ((\forall V3x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V0f\ V3x))\ V2t)))) \wedge (\exists V4g \in (A\_27a^{A\_27b}).((\forall V5x \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V5x)\ V2t)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ (ap\ V4g\ V5x))\ V1s)))) \wedge ((\forall V6x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V6x)\ V1s)) \Rightarrow ((ap\ V4g\ (ap\ V0f\ V6x)) = V6x))) \wedge (\forall V7x \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V7x)\ V2t)) \Rightarrow ((ap\ V0f\ (ap\ V4g\ V7x)) = V7x))))))))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))\tag{35}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))\tag{36}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))\tag{37}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p))))\tag{38}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q)))))\tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V0x)\ V1y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V2w))))\Rightarrow((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V2w)))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1y)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V2w))))))))\tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2z \in A\_27a.(\forall V3w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V0x)\ V1y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ A\_27a)\ V2z)\ V3w)))))\Leftrightarrow((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V0x)\ V2z))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V3w))))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V1y)\ V2z))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V3w))))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V0x)\ V1y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V3w)))))))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\forall V1b \in A\_27a.(\forall V2w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V0a)\ V1b))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V2w))))\Rightarrow((ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ A\_27a)\ V0a)\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ A\_27a)\ V1b)\ V2w)) = (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ A\_27a)\ V0a)\ V2w)))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ A\_27a)\ V0x)\ V1w)) = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V2y \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V2y)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V2y)\ V0x))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V1w))))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27b)\ V0w1)\ V1w2)) \Leftrightarrow (\exists V2f \in (A\_27b^{A\_27a}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27a\ A\_27b)\ V2f)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V0w1))\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27b)\ V1w2))) \wedge (\forall V3x \in A\_27a.(\forall V4y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V3x)\ V4y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V0w1)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27b)\ (ap\ V2f\ V3x))\ (ap\ V2f\ V4y)))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27b)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27b)\ V1w2)))))))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). (\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V2w3 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c).(((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27b\ A\_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27c)\ V0w1)\ V2w3)))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V2w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V3x \in A\_27a.(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V1w1))\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27b)\ V2w2))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V1w1))) \wedge (\forall V4x \in A\_27a.(\forall V5y \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V4x)\ V5y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V1w1)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27b)\ (ap\ V0f\ V4x))\ (ap\ V0f\ V5y)))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27b)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27b)\ V2w2))))))))) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EBIJ}\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ A\_27a)\ V3x)\ V1w1)))\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ A\_27b)\ (ap\ V0f\ V3x))\ V2w2))))))))) \tag{46}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). (\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V2w3 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c).(((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27a\ A\_27b)\ V0w1)\ V1w2)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27b\ A\_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27a\ A\_27c)\ V0w1)\ V2w3))))))$$