

thm_2Ewellorder_2Estrict_subset (TMZtPhEnPU1QpFgbqA848Hsyzri2ZBnKcJ6)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x)))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 4 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21 2) (\lambda V 0t \in 2.V 0t))$.

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. nonempty A 0 \Rightarrow \forall A 1. nonempty A 1 \Rightarrow nonempty (ty_2Epair_2Eprod A 0 A 1) \quad (1)$$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. nonempty A. 27a \Rightarrow \forall A. 27b. nonempty A. 27b \Rightarrow c_2Epair_2ESND A. 27a A. 27b \in (A. 27b^{(ty_2Epair_2Eprod A. 27a A. 27b)}) \quad (2)$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. nonempty A. 27a \Rightarrow \forall A. 27b. nonempty A. 27b \Rightarrow c_2Epair_2EFST A. 27a A. 27b \in (A. 27a^{(ty_2Epair_2Eprod A. 27a A. 27b)}) \quad (3)$$

Definition 5 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. if (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (ap V 0P (ap (c_2Emin_2E_40 (2^{A-27a}))$

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define `c_2Ebool_2EIN` to be $\lambda A. 27a : \iota. (\lambda V 0x \in A. 27a. (\lambda V 1f \in (2^{A-27a}). (ap V 1f V 0x)))$

Definition 9 We define `c_2Epred_set_2ESUBSET` to be $\lambda A. 27a : \iota. \lambda V 0s \in (2^{A-27a}). \lambda V 1t \in (2^{A-27a}). (ap (c_2Emin_2E_40 (2^{A-27a}))$

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.(ap (c_2Ebool_2E_7E V2t) c_2Ebool_2E_7E)))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (4)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (c_2Epair_2EABS_prod A_27a A_27b))$

Definition 13 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27b})^{A_27a}$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \end{aligned} \quad (5)$$

Definition 14 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \quad (15)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in A_27b. (((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in A_27b. (((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in (ty_2Epair_2Eprod A_27a A_27b). ((ap (ap (c_2Epair_2E_2C A_27a A_27b) (ap (c_2Epair_2EFST A_27a A_27b) V0x)) (ap (c_2Epair_2ESND A_27a A_27b) V0x)) = V0x)) \quad (20)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\forall V1x \in \\
& \quad A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\
& \quad A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\
& \quad (ap\ (ap\ V0f\ V1x)\ V2y))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). ((\forall V1p \in \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow (\forall V2p_1 \in \\
& \quad A_27a. (\forall V3p_2 \in A_27b. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A_27a\ A_27b)\ V2p_1)\ V3p_2))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{23}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0r1 \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). \\
& \quad (\forall V1r2 \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}). ((p\ (ap\ (ap \\
& \quad (c_2Epred_set_2ESUBSET\ (ty_2Epair_2Eprod\ A_27a\ A_27a))\ V0r1) \\
& \quad V1r2)) \Rightarrow (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27a))\ (ap\ (c_2Eset_relation_2Estrict\ A_27a)\ V0r1))\ (\\
& \quad ap\ (c_2Eset_relation_2Estrict\ A_27a)\ V1r2))))))
\end{aligned}$$