

thm_2Ewellorder_2Ewellorder__EMPTY
(TMVzFjCfEenSizQMkiUfWHX7TabZcNEAz9e)

October 26, 2020

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EFST\ A.27a\ A.27b \in (A.27a)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \quad (3)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$.

Definition 3 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$.

Definition 4 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2EF)$.

Definition 6 We define $c_2Epair_2EUNCURRY$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in ((A.27c)^{A-27b})$.

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epred_set_2EGSPEC\ A.27a\ A.27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod\ A.27a\ 2)^{A-27b}}) \quad (4)$$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (5)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).ap V1f V0x))$

Definition 12 We define $c_2Eset_relation_2Eantisym$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 13 We define $c_2Eset_relation_2Etransitive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 14 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) then (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 18 We define $c_2Eset_relation_2Ereflexive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 19 We define $c_2Eset_relation_2Erange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 20 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 21 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 22 We define $c_2Eset_relation_2Elinear_order$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 23 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 24 We define $c_2Ewellorder_2Ewellfounded$ to be $\lambda A_27a : \iota.\lambda V0R \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 25 We define $c_2Ewellorder_2Ewellorder$ to be $\lambda A_27a : \iota.\lambda V0R \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Assume the following.

$$True \quad (6)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (17)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ V0x)\ V1y) = (ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (18)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0x \in (ty_{.2Epair_2Eprod}\ A_{.27a}\ A_{.27b}).((ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ (ap\ (c_{.2Epair_2EFST}\ A_{.27a}\ A_{.27b})\ V0x))\ (ap\ (c_{.2Epair_2ESND}\ A_{.27a}\ A_{.27b})\ V0x)) = V0x)) \quad (19)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}.nonempty\ A_{.27c} \Rightarrow (\forall V0f \in ((A_{.27c}^{A_{.27b}})^{A_{.27a}}).(\forall V1x \in A_{.27a}.(\forall V2y \in A_{.27b}.((ap\ (ap\ (c_{.2Epair_2EUNCURRY}\ A_{.27a}\ A_{.27b}\ A_{.27c})\ V0f)\ (ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ A_{.27b})\ V1x)\ V2y))) = (ap\ (ap\ V0f\ V1x)\ V2y)))))) \quad (20)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27a}}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V2x)\ V1t))))))) \quad (21)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\forall V0f \in ((ty_{.2Epair_2Eprod}\ A_{.27a}\ 2)^{A_{.27b}}).(\forall V1v \in A_{.27a}.((p\ (ap\ (ap\ (c_{.2Ebool_2EIN}\ A_{.27a})\ V1v)\ (ap\ (c_{.2Epred_set_2EGSPEC}\ A_{.27a}\ A_{.27b})\ V0f))) \Leftrightarrow (\exists V2x \in A_{.27b}.((ap\ (ap\ (c_{.2Epair_2E_2C}\ A_{.27a}\ 2)\ V1v)\ c_{.2Ebool_2ET}) = (ap\ V0f\ V2x)))))) \quad (22)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V0x)\ (c_2Epred_set_2EEMPTY\ A_{.27a})))))) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ & (2^{A_{.27a}}).(\forall V2x \in A_{.27a}.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a}) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_{.27a})\ V0s)\ V1t)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a}) \\ & V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_{.27a})\ V2x)\ V1t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & ((\forall V0s \in (2^{A_{.27a}}).(\forall V1t \in \\ & (2^{A_{.27a}}).((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_{.27a})\ V0s)\ (ap\ (\\ & ap\ (c_2Epred_set_2EUNION\ A_{.27a})\ V0s)\ V1t)))))) \wedge (\forall V2s \in \\ & (2^{A_{.27a}}).(\forall V3t \in (2^{A_{.27a}}).((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET \\ & A_{.27a})\ V2s)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_{.27a})\ V3t)\ V2s)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & ((ap\ (c_2Epred_set_2EGSPEC\ A_{.27a} \\ & A_{.27a})\ (\lambda V0x \in A_{.27a}.(ap\ (ap\ (c_2Epair_2E_2C\ A_{.27a}\ 2)\ V0x)\ c_2Ebool_2EF))) = \\ & (c_2Epred_set_2EEMPTY\ A_{.27a})) \end{aligned} \quad (26)$$

Theorem 1

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (p\ (ap\ (c_2Ewellorder_2Ewellorder\ A_{.27a})\ (c_2Epred_set_2EEMPTY\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27a}))))$$