thm_2Ewellorder_2Ewellorder__fromNat__SUM (TMF4GDdMkq4a94z6vCcjST5hDoGefJnWSn5)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x$.

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{2}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Definition 8 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a:\iota.(\lambda V0P\in(2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a:\iota.(\lambda V0x\in A\_27a.(\lambda V1f\in(2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t\in$

Let $ty\_2Esum\_2Esum:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{4}$$

Let $c\_2Esum\_2EABS\_\_sum:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b\in((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{5}$$

Definition 13 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 14 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Enum\_2Enum:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

Let $c\_2Enum\_2EREP\_\_num:\iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num\in(omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2ESUC\_\_REP:\iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP\in(omega^{omega}) \tag{8}$$

Let $c\_2Enum\_2EABS\_\_num:\iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num\in(ty\_2Enum\_2Enum^{omega}) \tag{9}$$

Definition 15 *We define* c_2Enum_2ESUC *to be* $\lambda V0m\in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 16 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m\in ty\_2Enum\_2Enum.\lambda V1n\in ty\_2Enum\_2Enu$

Definition 17 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m\in ty\_2Enum\_2Enum.\lambda V1n\in ty\_2Enum\_2$

Let $c\_2Epair\_2EABS\_\_prod:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b\in((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 18 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Definition 19 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{11}$$

Definition 20 *We define* c_2Epair_2E_23_23 *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f \in (A\_27$

Definition 21 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 22 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A}$

Definition 23 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

Definition 24 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 25 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 26 *We define* c_2Eset__relation_2Ereflexive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Definition 27 *We define* c_2Eset__relation_2Eantisym *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Definition 28 *We define* c_2Eset__relation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_}$

Definition 29 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 30 *We define* c_2Eset__relation_2Elinear__order *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a}$

Definition 31 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 32 *We define* c_2Ewellorder_2Ewellfounded *to be* $\lambda A\_27a : \iota.\lambda V0R \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_}$

Definition 33 *We define* c_2Ewellorder_2Ewellorder *to be* $\lambda A\_27a : \iota.\lambda V0R \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a}$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{20}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a)\wedge(p\ (ap\ V0P\ V2x))))\Leftrightarrow(p\ (ap\ V0P\ V1a))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b))\Leftrightarrow((V0x = V2a)\wedge(V1y = V3b))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)){\Leftrightarrow}((V0x = V2a){\wedge}(V1y = V3b))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0x))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V0x)) = V0x)) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c. nonempty\ A\_27c{\Rightarrow}(\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) = (ap\ (ap\ V0f\ V1x)\ V2y))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0P \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}).((\exists V1p \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(p\ (ap\ V0P\ V1p))){\Leftrightarrow}(\exists V2p\_\_1 \in A\_27a.(\exists V3p\_\_2 \in A\_27b.(p\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2p\_\_1)\ V3p\_\_2))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c. nonempty\ A\_27c{\Rightarrow}\forall A\_27d.nonempty\ A\_27d{\Rightarrow}(\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27d^{A\_27c}).(\forall V2x \in A\_27a.(\forall V3y \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ A\_27a\ A\_27c\ A\_27b\ A\_27d)\ V0f)\ V1g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27c)\ V2x)\ V3y)) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27d)\ (ap\ V0f\ V2x))\ (ap\ V1g\ V3y))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t){\Leftrightarrow}(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)){\Leftrightarrow}(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \lor (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V1s)) \Rightarrow (\forall V2f \in (A\_27b^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V2f\ V0x))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0y \in A\_27a.(\forall V1x \in A\_27a.(((ap\ (\mathsf{c\_2Esum\_2EINL}\ A\_27a\ A\_27b)\ V1x) = (ap\ (\mathsf{c\_2Esum\_2EINL}\ A\_27a\ A\_27b)\ V0y)) \Leftrightarrow (V1x = V0y)))) \land (\forall V2y \in A\_27b.(\forall V3x \in A\_27b.(((ap\ (\mathsf{c\_2Esum\_2EINR}\ A\_27a\ A\_27b)\ V3x) = (ap\ (\mathsf{c\_2Esum\_2EINR}\ A\_27a\ A\_27b)\ V2y)) \Leftrightarrow (V3x = V2y))))) \tag{34}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewellorder}\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ 2))\ (\lambda V1i \in ty\_2Enum\_2Enum.(\lambda V2j \in ty\_2Enum\_2Enum.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ 2)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1i)\ V2j))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1i)\ V2j))\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2j)\ V0n))))))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2t \in (2^{A\_27b}).(((p\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewellorder}\ A\_27a)\ V0r)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINJ}\ A\_27a\ A\_27b)\ V1f)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}\ A\_27a\ A\_27a)\ V0r))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27a\ A\_27a)\ V0r)))\ V2t))) \Rightarrow (p\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewellorder}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_23\_23}\ A\_27a\ A\_27a\ A\_27b\ A\_27b)\ V1f)\ V1f))\ V0r))))))) \tag{36}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewellorder}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ 2))\ (\lambda V1i \in ty\_2Enum\_2Enum.(\lambda V2j \in ty\_2Enum\_2Enum.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Epair\_2Eprod\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ 2)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ (ap\ (\mathsf{c\_2Esum\_2EINL}\ ty\_2Enum\_2Enum\ A\_27a)\ V1i))\ (ap\ (\mathsf{c\_2Esum\_2EINL}\ ty\_2Enum\_2Enum\ A\_27a)\ V2j)))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1i)\ V2j))\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2j)\ V0n)))))))))))$$