





**Definition 21** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c$

**Definition 22** We define  $c\_2Ewellorder\_2EelsOf$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder A\_27a$

**Definition 23** We define  $c\_2Eset\_relation\_2Erestrict$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a$

**Definition 24** We define  $c\_2Eset\_relation\_2Ereflexive$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a$

**Definition 25** We define  $c\_2Eset\_relation\_2Eantisym$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a$

**Definition 26** We define  $c\_2Eset\_relation\_2Etransitive$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a$

**Definition 27** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap (c$

**Definition 28** We define  $c\_2Eset\_relation\_2Elinear\_order$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a$

**Definition 29** We define  $c\_2Eset\_relation\_2Estrict$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a$

**Definition 30** We define  $c\_2Ewellorder\_2Ewellfounded$  to be  $\lambda A\_27a : \iota.\lambda V0R \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a$

**Definition 31** We define  $c\_2Ewellorder\_2Ewellorder$  to be  $\lambda A\_27a : \iota.\lambda V0R \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (13)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1a \in A.27a.((\exists V2x \in A.27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in A.27b.(((ap (ap (c.2Epair_2E_2C A.27a A.27b) V0x) V1y) = (ap (ap (c.2Epair_2E_2C A.27a A.27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in A.27b.(((ap (ap (c.2Epair_2E_2C A.27a A.27b) V0x) V1y) = (ap (ap (c.2Epair_2E_2C A.27a A.27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod A.27a A.27b).((ap (ap (c.2Epair_2E_2C A.27a A.27b) (ap (c.2Epair_2EFST A.27a A.27b) V0x)) (ap (c.2Epair_2ESND A.27a A.27b) V0x)) = V0x)) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c.nonempty A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}).(\forall V1x \in A.27a.(\forall V2y \in A.27b.((ap (ap (c.2Epair_2EUNCURRY A.27a A.27b A.27c) V0f) (ap (ap (c.2Epair_2E_2C A.27a A.27b) V1x) V2y)) = (ap (ap V0f V1x) V2y)))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0P \in (2^{(ty\_2Epair\_2Eprod A.27a A.27b)}).((\forall V1p \in (ty\_2Epair\_2Eprod A.27a A.27b).(p (ap V0P V1p))) \Leftrightarrow (\forall V2p_{-1} \in A.27a.(\forall V3p_{-2} \in A.27b.(p (ap V0P (ap (ap (c.2Epair_2E_2C A.27a A.27b) V2p_{-1}) V3p_{-2})))))) \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ & (2^{A\_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}). (\forall V1v \in \\ & A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap\ (ap\ (c\_2Epair\_2E\_2C \\ & A\_27a\ 2)\ V1v)\ c\_2Ebool\_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1x \in \\ & A\_27a. (\forall V2y \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V1x) \\ & (ap\ (ap\ (c\_2Epred\_set\_2EDELETE\ A\_27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap \\ & (c\_2Ebool\_2EIN\ A\_27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y)))))) \end{aligned} \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B)) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False) \Rightarrow False)) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg( \\ & p\ V2r)) \vee (\neg(p\ V1q))) \wedge ((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{35}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. \text{nonempty } A\_27a \Rightarrow ((\forall V0a \in (ty\_2Ewellorder\_2Ewellorder \\
& A\_27a). ((ap (c\_2Ewellorder\_2Ewellorder\_ABS A\_27a) (ap (c\_2Ewellorder\_2Ewellorder\_REP \\
& A\_27a) V0a)) = V0a)) \wedge (\forall V1r \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a)}). \\
& ((p (ap (c\_2Ewellorder\_2Ewellorder A\_27a) V1r)) \Leftrightarrow ((ap (c\_2Ewellorder\_2Ewellorder\_REP \\
& A\_27a) (ap (c\_2Ewellorder\_2Ewellorder\_ABS A\_27a) V1r)) = V1r))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in \\
& A\_27a. (\forall V2w \in (ty\_2Ewellorder\_2Ewellorder A\_27a). ((p \\
& (ap (ap (c\_2Ebool\_2EIN (ty\_2Epair\_2Eprod A\_27a A\_27a)) (ap (ap \\
& (c\_2Epair\_2E\_2C A\_27a A\_27a) V0x) V1y)) (ap (c\_2Ewellorder\_2Ewellorder\_REP \\
& A\_27a) V2w))) \Rightarrow ((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V0x) (ap (c\_2Ewellorder\_2EelsOf \\
& A\_27a) V2w))) \wedge (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V1y) (ap (c\_2Ewellorder\_2EelsOf \\
& A\_27a) V2w)))))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0w \in (ty\_2Ewellorder\_2Ewellorder \\
& A\_27a). (\forall V1s \in (2^{A\_27a}). (p (ap (c\_2Ewellorder\_2Ewellorder \\
& A\_27a) (ap (ap (c\_2Eset\_relation\_2Errestrict A\_27a) (ap (c\_2Ewellorder\_2Ewellorder\_REP \\
& A\_27a) V0w)) V1s))))))
\end{aligned} \tag{40}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0e \in A\_27a. (\forall V1w \in \\ & (ty\_2Ewellorder\_2Ewellorder\ A\_27a). (p\ (ap\ (c\_2Ewellorder\_2Ewellorder \\ & A\_27a)\ (ap\ (c\_2Epred\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a) \\ & (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (c\_2Epair\_2EUNCURRY\ A\_27a \\ & A\_27a\ (ty\_2Epair\_2Eprod\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)\ 2))\ ( \\ & \lambda V2x \in A\_27a. (\lambda V3y \in A\_27a. (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Epair\_2Eprod \\ & A\_27a\ A\_27a)\ 2)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a\ A\_27a)\ V2x)\ V3y)) \\ & (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ c\_2Ebool\_2E\_7E\ (ap\ (ap\ (c\_2Emin\_2E\_3D \\ & A\_27a)\ V2x)\ V0e)))\ (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ c\_2Ebool\_2E\_7E \\ & (ap\ (ap\ (c\_2Emin\_2E\_3D\ A\_27a)\ V3y)\ V0e)))\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27a \\ & A\_27a)\ V2x)\ V3y))\ (ap\ (c\_2Ewellorder\_2Ewellorder\_REP\ A\_27a) \\ & V1w))))))))))))) \end{aligned}$$