thm_2Ewellorder_2Ewellorder__rrestrict (TMQj138iVCjskMr3Nb4aJ4egn9fe5vH33YU)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (1)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (2)$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 12 *We define* c_2Ewellorder_2Ewellfounded *to be* $\lambda A\_27a : \iota.\lambda V0R \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 13 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A$

Definition 14 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod$

Definition 15 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 16 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 17 *We define* c_2Eset__relation_2Ereflexive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2$

Definition 18 *We define* c_2Eset__relation_2Eantisym *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27$

Definition 19 *We define* c_2Eset__relation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_$

Definition 20 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 21 *We define* c_2Eset__relation_2Elinear__order *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{4}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{5}$$

Definition 22 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 23 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 24 *We define* c_2Ewellorder_2Ewellorder *to be* $\lambda A\_27a : \iota.\lambda V0R \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a$

Let $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{6}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \tag{7}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{8}$$

Definition 25 *We define* $\mathsf{c\_2Eset\_\_relation\_2Errestrict}$ *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{10}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg (p\ V0A)) \Rightarrow False))) \tag{14}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg (p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{15}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{16}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg (p\ V1q))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V1s \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s))\ V0r)))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r0 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(((p\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewellfounded}\ A\_27a)\ V1r)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ V0r0)\ V1r))) \Rightarrow (p\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewellfounded}\ A\_27a)\ V0r0))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((ap\ (c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). ((p\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewellorder}\ A\_27a)\ V1r)) \Leftrightarrow ((ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a)\ V1r)) = V1r)))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r1 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1r2 \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ V0r1)\ V1r2)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ V0r1))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ V1r2)))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}). (\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ V0r)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}\ A\_27a\ A\_27a)\ V0r))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27a\ A\_27a)\ V0r)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Elinear\_\_order}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}\ A\_27a\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s)))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27a\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s)))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}).(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereflexive}\ A\_27a)\ V0r)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}\ A\_27a\ A\_27a)\ V0r))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27a\ A\_27a)\ V0r)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Ereflexive}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Edomain}\ A\_27a\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s)))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Erange}\ A\_27a\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ V0r)\ V1s)))))))) \quad (23)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1s \in (2^{A\_27a}).(p\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewellorder}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Errestrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V0w))\ V1s)))))$$