

thm_2Ewellorder_2Ewo2wo__EQ__SOME__downwards (TMZngvFknTXSfN3g2LvDpcPjH6DUeqv8H7y)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 7 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (5)$$

Definition 8 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$.

Definition 9 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ A_27a)\ V0P)))$.

Definition 10 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$.

Definition 12 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E2F))$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (7)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (9)$$

Definition 13 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$.

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$.

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (10)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (11)$$

Definition 15 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a})$.
 Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(12)

Definition 16 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$.

Definition 17 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$.

Definition 18 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS\ A_27a\ A_27b))$.

Definition 19 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a))$.

Definition 20 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 21 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27b)$.

Definition 22 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 23 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Einsert_2E_A_27a\ x\ s))$.

Definition 24 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ediff_2E_A_27a\ s\ t))$.

Definition 25 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap\ (ap\ (c_2Edelete_2E_A_27a\ x\ s)))$.

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2ETHE\ A_27a \in (A_27a^{(ty_2Eoption_2Eoption\ A_27a)})$$
(13)

Definition 26 We define $c_2Eset_relation_2Erange$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$.

Definition 27 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$.

Definition 28 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Eunion_2E_A_27a\ s\ t))$.

Definition 29 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 30 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_2Ebool_2ECOND\ t1\ t2\ t3))))$.

Definition 31 We define $c_2Eoption_2Esome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ P\ (c_2Eoption_2Eoption_ABS\ A_27a))))$.

Definition 32 We define $c_2Ewellorder_2Ewleast$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 33 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27a. (c_2Ebool_2ELET\ f\ x)))$.

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a$$
(14)

Definition 34 We define $c_2\text{Erelation_2ERESTRICT}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1f$

Definition 35 We define $c_2\text{Erelation_2ETC}$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1a \in A_27a.\lambda V2b$

Definition 36 We define $c_2\text{Erelation_2Eapprox}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 37 We define $c_2\text{Erelation_2Ethe_fun}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 38 We define $c_2\text{Erelation_2EWFREC}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1M$

Definition 39 We define $c_2\text{Ewellorder_2Ewo2wo}$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2\text{Ewellorder_2E}$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \tag{17}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \tag{19}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \tag{21}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V1P V3x))) \vee (p V0Q)))))) \tag{22}$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \ A_{27a}). ((\exists V1x \in A_{27a}. (V0opt = (ap \ (c_2Eoption_2ESOME \ A_{27a} \ V1x))) \vee (V0opt = (c_2Eoption_2ENONE \ A_{27a})))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. ((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (30)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))))) \quad (31)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in \\ & (ty_2Ewellorder_2Ewellorder\ A_27b).(\forall V2x \in A_27a.(((\\ & ap\ (ap\ (ap\ (c_2Ewellorder_2Ewo2wo\ A_27a\ A_27b)\ V0w1)\ V1w2)\ V2x) = \\ & (c_2Eoption_2ENONE\ A_27b)) \Rightarrow (\forall V3y \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & (ty_2Epair_2Eprod\ A_27a\ A_27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\ & A_27a)\ V2x)\ V3y))\ (ap\ (c_2Eset_relation_2Estrict\ A_27a)\ (ap\ (\\ & c_2Ewellorder_2Ewellorder_REP\ A_27a)\ V0w1)))))) \Rightarrow ((ap\ (ap\ (ap \\ & (c_2Ewellorder_2Ewo2wo\ A_27a\ A_27b)\ V0w1)\ V1w2)\ V3y) = (c_2Eoption_2ENONE \\ & A_27b))))))))) \end{aligned} \quad (38)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in \\ & (ty_2Ewellorder_2Ewellorder\ A_27b).(\forall V2x \in A_27a.(\forall V3y \in \\ & A_27b.(((ap\ (ap\ (ap\ (c_2Ewellorder_2Ewo2wo\ A_27a\ A_27b)\ V0w1) \\ & V1w2)\ V2x) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V3y)) \Rightarrow (\forall V4x0 \in \\ & A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\ & (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V4x0)\ V2x))\ (ap\ (c_2Eset_relation_2Estrict \\ & A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A_27a)\ V0w1)))))) \Rightarrow \\ & (\exists V5y0 \in A_27b.((ap\ (ap\ (ap\ (c_2Ewellorder_2Ewo2wo\ A_27a \\ & A_27b)\ V0w1)\ V1w2)\ V4x0) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V5y0))))))))) \end{aligned}$$