thm_2Ewellorder_2Ewo2wo__mono (TM-RQaBReiw9G3CoPVcWG8qo48krgDdvUCNt)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{1}$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{2}$$

Let $c\_2Eoption\_2EIS\_\_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_NONE\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{3}$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Definition 10 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{4}$$

Definition 11 *We define* $\mathsf{c\_2Eone\_2Eone}$ *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{5}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{6}$$

Definition 12 *We define* $\mathsf{c\_2Esum\_2EINR}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{7}$$

Definition 13 *We define* $\mathsf{c\_2Eoption\_2ENONE}$ *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{8}$$

Let $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{9}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \tag{10}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{11}$$

Definition 14 *We define* $\mathsf{c\_2Epair\_2E\_2C}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 15 *We define* $\mathsf{c\_2Ebool\_2EIN}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{12}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{13}$$

Definition 16 *We define* $\mathsf{c\_2Epair\_2EUNCURRY}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{14}$$

Definition 17 *We define* $\mathsf{c\_2Eset\_\_relation\_2Estrict}$ *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 18 *We define* $\mathsf{c\_2Ewellorder\_2Eiseg}$ *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_2$

Definition 19 *We define* $\mathsf{c\_2Epred\_\_set\_2EEMPTY}$ *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 20 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 21 *We define* $\mathsf{c\_2Epred\_\_set\_2EINSERT}$ *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 22 *We define* $\mathsf{c\_2Epred\_\_set\_2EDIFF}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 23 *We define* $\mathsf{c\_2Epred\_\_set\_2EDELETE}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{15}$$

Definition 24 *We define* $\mathsf{c\_2Epred\_\_set\_2EIMAGE}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 25 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 26 *We define* $\mathsf{c\_2Eset\_\_relation\_2Erange}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A}$

Definition 27 *We define* $\mathsf{c\_2Eset\_\_relation\_2Edomain}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

Definition 28 *We define* $\mathsf{c\_2Epred\_\_set\_2EUNION}$ *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 29 *We define* $\mathsf{c\_2Ewellorder\_2EelsOf}$ *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A.$

Definition 30 *We define* $\mathsf{c\_2Esum\_2EINL}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 31 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 32 *We define* c_2Eoption_2Esome *to be* $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (ap\ ($c_2Ebool_2EC

Definition 33 *We define* c_2Ewellorder_2Ewleast *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{16}$$

Definition 34 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 35 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 36 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 37 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Definition 38 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Definition 39 *We define* c_2Ewellorder_2Ewo2wo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2E$

Definition 40 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Assume the following.

$$True \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{19}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{23}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{25}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1X \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V2x \in A\_27a.((((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0P)\ V1X)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \Leftrightarrow ((p\ V0P) \Rightarrow (p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ V1X)))) \wedge ((((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0P)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V1X) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \Leftrightarrow ((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V1X)) \Rightarrow (p\ V0P))) \wedge ((((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0P)\ V1X)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x)) \Leftrightarrow ((p\ V0P) \wedge (V1X = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x)))) \wedge (((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0P)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V1X) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1X = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))))))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{35}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{37}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{38}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{39}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V0p))))) \tag{40}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p))) \Rightarrow (p\ V0p))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V0x)\ V1y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V2w)))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V2w))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1y)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V2w)))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V0x)\ V0x))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V1w)))) \Leftrightarrow False))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V2x \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewo2wo}\ A\_27a\ A\_27b)\ V0w)\ V1w2)\ V2x) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ (2^{(ty\_2Eoption\_2Eoption\ A\_27b)})\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (\lambda V3s0 \in (2^{(ty\_2Eoption\_2Eoption\ A\_27b)}).(ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ (2^{A\_27b})\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (\lambda V4s1 \in (2^{A\_27b}).(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27b}))\ V4s1)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27b)\ V1w2)))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewleast}\ A\_27b)\ V1w2)\ V4s1))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eoption\_2Eoption\ A\_27b)\ A\_27b)\ (c\_2Eoption\_2ETHE\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ (ty\_2Eoption\_2Eoption\ A\_27b))\ V3s0)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewo2wo}\ A\_27a\ A\_27b)\ V0w)\ V1w2))\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eiseg}\ A\_27a)\ V0w)\ V2x))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x1 \in A\_27a.(\forall V1x2 \in A\_27a.(\forall V2w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V3w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V0x1)\ V1x2))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V2w1)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eoption\_2Eoption\ A\_27b)\ A\_27b)\ (c\_2Eoption\_2ETHE\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewo2wo}\ A\_27a\ A\_27b)\ V2w1)\ V3w2))\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eiseg}\ A\_27a)\ V2w1)\ V0x1)))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eoption\_2Eoption\ A\_27b)\ A\_27b)\ (c\_2Eoption\_2ETHE\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EDELETE}\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewo2wo}\ A\_27a\ A\_27b)\ V2w1)\ V3w2))\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eiseg}\ A\_27a)\ V2w1)\ V1x2)))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))))))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x1 \in A\_27a.(\forall V1w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V2x2 \in A\_27a.(\forall V3w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V4y \in A\_27b.(((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x1)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V1w1))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x2)\ (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ V1w1))) \wedge (((ap\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewo2wo}\ A\_27a\ A\_27b)\ V1w1)\ V3w2)\ V0x1) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V4y)) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewo2wo}\ A\_27a\ A\_27b)\ V1w1)\ V3w2)\ V2x2) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V4y))))) \Rightarrow (V0x1 = V2x2))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1s1 \in (2^{A\_27a}).(\forall V2x \in A\_27a.(\forall V3s2 \in (2^{A\_27a}).(\forall V4y \in A\_27a.((((ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewleast}\ A\_27a)\ V0w)\ V1s1) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x)) \wedge (((ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewleast}\ A\_27a)\ V0w)\ V3s2) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V4y)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1s1)\ V3s2)))) \Rightarrow ((V2x = V4y) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V2x)\ V4y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V0w))))))))))) \tag{48}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V2x0 \in A\_27b.(\forall V3y0 \in A\_27a.(\forall V4x \in A\_27b.(\forall V5y \in A\_27a.((((ap\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewo2wo}\ A\_27b\ A\_27a)\ V0w1)\ V1w2)\ V2x0) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3y0)) \wedge (((ap\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewo2wo}\ A\_27b\ A\_27a)\ V0w1)\ V1w2)\ V4x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V5y)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27b)\ V2x0)\ V4x))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27b)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27b)\ V0w1)))))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V3y0)\ V5y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V1w2)))))))))))$$