

thm_2Ewellorder_2Ewobound2
(TMJtT24vqYn3X8iYvnbqBAf72nC6nmprAq5)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o(x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o(p \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 10 We define $c_2Ebool_2E_EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (3)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Definition 11 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (5)$$

Definition 12 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP \\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \end{aligned} \quad (7)$$

Definition 13 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 14 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (8)$$

Definition 15 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellord$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ & A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \end{aligned} \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (25)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (30)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.(\forall V2w \in (ty_2Ewellorder_2Ewellorder A.27a).(\forall V3z \in \\ & A.27a.(((p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A.27a A.27a)) \\ (ap (ap (c_2Epair_2E_2C A.27a A.27a) V0x) V1y)) (ap (c_2Eset_relation_2Estrict \\ A.27a) (ap (c_2Ewellorder_2Ewellorder_REP A.27a) V2w)))))) \wedge (\\ p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A.27a A.27a)) (ap (ap \\ (c_2Epair_2E_2C A.27a A.27a) V1y) V3z)) (ap (c_2Eset_relation_2Estrict \\ A.27a) (ap (c_2Ewellorder_2Ewellorder_REP A.27a) V2w)))))) \Rightarrow \\ & (p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A.27a A.27a)) (ap (\\ ap (c_2Epair_2E_2C A.27a A.27a) V0x) V3z)) (ap (c_2Eset_relation_2Estrict \\ A.27a) (ap (c_2Ewellorder_2Ewellorder_REP A.27a) V2w))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.(\forall V2z \in A.27a.(\forall V3w \in (ty_2Ewellorder_2Ewellorder \\ & A.27a).((p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A.27a A.27a)) \\ (ap (ap (c_2Epair_2E_2C A.27a A.27a) V0x) V1y)) (ap (c_2Eset_relation_2Estrict \\ A.27a) (ap (c_2Ewellorder_2Ewellorder_REP A.27a) (ap (ap (c_2Ewellorder_2Ewobound \\ A.27a) V2z) V3w)))))) \Leftrightarrow ((p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod \\ A.27a A.27a)) (ap (ap (c_2Epair_2E_2C A.27a A.27a) V0x) V2z)) (ap \\ (c_2Eset_relation_2Estrict A.27a) (ap (c_2Ewellorder_2Ewellorder_REP \\ A.27a) V3w)))))) \wedge ((p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A.27a \\ A.27a)) (ap (ap (c_2Epair_2E_2C A.27a A.27a) V1y) V2z)) (ap (c_2Eset_relation_2Estrict \\ A.27a) (ap (c_2Ewellorder_2Ewellorder_REP A.27a) V3w)))))) \wedge (\\ p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A.27a A.27a)) (ap (ap \\ (c_2Epair_2E_2C A.27a A.27a) V0x) V1y)) (ap (c_2Eset_relation_2Estrict \\ A.27a) (ap (c_2Ewellorder_2Ewellorder_REP A.27a) V3w))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& \quad A.27a. (\forall V2z \in A.27a. (\forall V3w \in (ty_2Ewellorder_2Ewellorder \\
& \quad A.27a). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27a\ A.27a)) \\
& (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V0x)\ V1y))\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\
& \quad A.27a)\ (ap\ (ap\ (c_2Ewellorder_2Ewobound\ A.27a)\ V2z)\ V3w)))) \Leftrightarrow (\\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27a\ A.27a))\ (ap\ (\\
& \quad ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V0x)\ V2z))\ (ap\ (c_2Eset_relation_2Estrict \\
& \quad A.27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A.27a)\ V3w)))) \wedge (\\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27a\ A.27a))\ (ap\ (\\
& \quad ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V1y)\ V2z))\ (ap\ (c_2Eset_relation_2Estrict \\
& \quad A.27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A.27a)\ V3w)))) \wedge (\\
& \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27a\ A.27a))\ (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A.27a\ A.27a)\ V0x)\ V1y))\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\
& \quad A.27a)\ V3w)))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder \\
& \quad A.27a). (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A.27a). (\\
& \quad (V0w1 = V1w2) \Leftrightarrow (\forall V2a \in A.27a. (\forall V3b \in A.27a. ((p\ (ap\ (\\
& \quad ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27a\ A.27a))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A.27a\ A.27a)\ V2a)\ V3b))\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A.27a) \\
& \quad V0w1))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27a\ A.27a)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V2a)\ V3b))\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\
& \quad A.27a)\ V1w2)))))))))
\end{aligned} \tag{35}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in A.27a. (\forall V1b \in \\
& \quad A.27a. (\forall V2w \in (ty_2Ewellorder_2Ewellorder\ A.27a). ((p \\
& \quad (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27a\ A.27a))\ (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A.27a\ A.27a)\ V0a)\ V1b))\ (ap\ (c_2Eset_relation_2Estrict \\
& \quad A.27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A.27a)\ V2w)))) \Rightarrow (\\
& \quad (ap\ (ap\ (c_2Ewellorder_2Ewobound\ A.27a)\ V0a)\ (ap\ (ap\ (c_2Ewellorder_2Ewobound \\
& \quad A.27a)\ V1b)\ V2w)) = (ap\ (ap\ (c_2Ewellorder_2Ewobound\ A.27a)\ V0a) \\
& \quad V2w))))))
\end{aligned}$$