

thm_2Ewhile_2EOWHILE__EQ__NONE (TM- RvMq12XuaiNKsRFW3xz4xvPJYBtFqCbC)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A-27b})^{A-27a})^2}) \tag{3}$$

Definition 10 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eone_2Eone))$
Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (6)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 12 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (10)$$

Definition 13 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num m)$

Definition 14 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g \in (A_27a^{A_27c}).$

Let $c_2Ewhile_2EWHILE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ewhile_2EWHILE A_27a \in (((A_27a^{A_27a})^{(A_27a^{A_27a})})^{(2^{A_27a})}) \quad (11)$$

Definition 15 We define $c_2Ewhile_2ELEAST$ to be $\lambda V0P \in (2^{ty_2Enum_2Enum}). (ap (ap (ap (c_2Ewhile_2EWHILE P))))$

Definition 16 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS A_27a e))$

Definition 17 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption A_27a x))$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Earithmetic_2EFUNPOW\ A_27a \in \left(((A_27a^{A_27a})_{ty_2Enum_2Enum})_{(A_27a^{A_27a})} \right) \quad (12)$$

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 20 We define $c_2Ewhile_2EOWHILE$ to be $\lambda A_27a : \iota. \lambda V0G \in (2^{A_27a}). \lambda V1f \in (A_27a^{A_27a}). \lambda V$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\left((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \right) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x))))))) \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1x \in A_27a. \\ & (\forall V2y \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x))\ (c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (\neg(p\ V0P)))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (p\ V0P)) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x))\ (c_2Eoption_2ENONE\ A_27a)) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y)))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (c_2Eoption_2ENONE\ A_27a))\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = V2y))))))))) \quad (17) \end{aligned}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0G \in (2^{A_27a}). (\forall V1f \in (A_27a^{A_27a}). (\forall V2s \in A_27a. (((ap\ (ap\ (ap\ (c_2Ewhile_2EOWHILE\ A_27a)\ V0G)\ V1f)\ V2s) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (\forall V3n \in ty_2Enum_2Enum. (p\ (ap\ V0G\ (ap\ (ap\ (ap\ (c_2Earithmetic_2EFUNPOW\ A_27a)\ V1f)\ V3n)\ V2s))))))))))$$