

thm_2Ewhile_2EOWHILE__IND
(TMGJ1FTX2WxnkNhJSkEx1v1Q1uoPbDFdPY8)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (6)$$

Definition 14 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (7)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (8)$$

Definition 15 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (9)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (10)$$

Definition 16 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c$

Definition 17 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in$

Let $c_2Ewhile_2EWHILE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewhile_2EWHILE\ A_27a \in (((A_27a^{A_27a})^{(A_27a^{A_27a})})^{(2^{A_27a})}) \quad (11)$$

Definition 18 We define $c_2Ewhile_2ELEAST$ to be $\lambda V0P \in (2^{ty_2Enum_2Enum}).(ap\ (ap\ (ap\ (c_2Ewhile_2$

Definition 19 We define c_Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_Esum_2EABS$

Definition 20 We define $c_Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_Eoption_2Eoption_2$

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Earithmetic_2EFUNPOW A_27a \in ((A_27a^{A_27a})^{ty_2Enum_2Enum})^{(A_27a^{A_27a})} \quad (12)$$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda$

Definition 22 We define $c_2Ewhile_2EOWHILE$ to be $\lambda A_27a : \iota.\lambda V0G \in (2^{A_27a}).\lambda V1f \in (A_27a^{A_27a}).\lambda V$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow ((\forall V0f \in (A_27a^{A_27a}).(\forall V1x \in \\ & A_27a.((ap (ap (ap (c_2Earithmetic_2EFUNPOW A_27a) V0f) c_2Enum_2E0 \\ & V1x) = V1x))) \wedge (\forall V2f \in (A_27a^{A_27a}).(\forall V3n \in ty_2Enum_2Enum. \\ & (\forall V4x \in A_27a.((ap (ap (ap (c_2Earithmetic_2EFUNPOW A_27a) \\ & V2f) (ap c_2Enum_2ESUC V3n)) V4x) = (ap (ap (ap (c_2Earithmetic_2EFUNPOW \\ & A_27a) V2f) V3n) (ap V2f V4x)))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (27)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (28)$$

Assume the following.

$$\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}). (\forall V1v \in A_27a. ((\forall V2x \in A_27a. ((V2x = V1v) \Rightarrow (p (ap\ V0f\ V2x)))) \Leftrightarrow (p (ap\ V0f\ V1v)))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. (((\exists V2x \in A_27a. (p (ap\ V0P\ V2x))) \Rightarrow (p\ V1Q)) \Leftrightarrow (\forall V3x \in A_27a. ((p (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \wedge (((\exists V4x \in A_27a. (p (ap\ V0P\ V4x))) \wedge (p\ V1Q)) \Leftrightarrow (\exists V5x \in A_27a. ((p (ap\ V0P\ V5x)) \wedge (p\ V1Q)))))) \wedge (((p\ V1Q) \wedge (\exists V6x \in A_27a. (p (ap\ V0P\ V6x)))) \Leftrightarrow (\exists V7x \in A_27a. ((p\ V1Q) \wedge (p (ap\ V0P\ V7x))))))) \quad (31)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}). (((p (ap\ V0P\ c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((p (ap\ V0P\ V1n)) \Rightarrow (p (ap\ V0P\ (ap\ c_2Enum_2ESUC\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p (ap\ V0P\ V2n)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. (((ap (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap (c_2Eoption_2ESOME\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE\ A_27a) = (ap (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \quad (34)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0) (ap\ c_2Enum_2ESUC\ V0n)))) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap (ap\ c_2Eprim_rec_2E_3C\ V0m)\ V1n)) \Rightarrow (p (ap (ap (ap\ c_2Eprim_rec_2E_3C\ (ap\ c_2Enum_2ESUC\ V0m))\ (ap\ c_2Enum_2ESUC\ V1n)))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in (2^{ty_2Enum_2Enum}). ((\exists V1n \in ty_2Enum_2Enum. (p (ap\ V0p\ V1n))) \Rightarrow ((p (ap\ V0p\ (ap\ c_2Ewhile_2ELEAST\ V0p))) \wedge (\forall V2n \in ty_2Enum_2Enum. ((p (ap (ap\ c_2Eprim_rec_2E_3C\ V2n)\ (ap\ c_2Ewhile_2ELEAST\ V0p))) \Rightarrow (\neg(p (ap\ V0p\ V2n)))))))) \quad (37)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in ((2^{A_27a})^{A_27a}). \\ & (\forall V1G \in (2^{A_27a}). (\forall V2f \in (A_27a^{A_27a}). (((\forall V3s \in \\ & A_27a. ((\neg(p\ (ap\ V1G\ V3s))) \Rightarrow (p\ (ap\ (ap\ V0P\ V3s)\ V3s)))) \wedge (\forall V4s1 \in \\ & A_27a. (\forall V5s2 \in A_27a. (((p\ (ap\ V1G\ V4s1)) \wedge (p\ (ap\ (ap\ V0P\ (ap \\ & V2f\ V4s1))\ V5s2))) \Rightarrow (p\ (ap\ (ap\ V0P\ V4s1)\ V5s2)))))) \Rightarrow (\forall V6s1 \in \\ & A_27a. (\forall V7s2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ewhile_2EOWHILE \\ & A_27a)\ V1G)\ V2f)\ V6s1) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V7s2)) \Rightarrow (\\ & p\ (ap\ (ap\ V0P\ V6s1)\ V7s2)))))))))) \end{aligned}$$