thm_2Ewords_2EADD__WITH__CARRY__SUB (TMGWkg-MMa5bPkVnEJRpM4BWqLUBHtugK61q)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{6}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 7 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 9 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Definition 10 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 11 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 12 *We define* c_2Emarker_2EAbbrev *to be* $\lambda V0x \in 2.V0x.$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 13 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t)).$

Definition 14 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 16 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 17 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 18 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 19 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 20 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 21 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 2$)\ (\lambda V2t \in$

Definition 22 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 23 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 24 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 25 *We define* c_2Enumeral_2EiiSUC *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap$ c_2Enum_2ESUC $(ap$

Definition 26 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 27 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{14}$$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{15}$$

Let $c\_2Ewords\_2Eadd\_\_with\_\_carry : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Eadd\_\_with\_\_carry\ A\_27a \in ((ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2))^{(ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2))}) \tag{16}$$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{17}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{18}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (19)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (20)$$

Definition 28 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 29 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap$ (c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \quad (21)$$

Definition 30 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Definition 31 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (22)$$

Definition 32 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap$

Let $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (23)$$

Definition 33 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 34 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 35 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 36 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 37 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Definition 38 *We define* c_2Ewords_2Eword__add *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V$

Definition 39 *We define* c_2Ewords_2Eword__sub *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{24}$$

Definition 40 *We define* $\mathsf{c\_2Epair\_2E\_2C}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 41 *We define* $\mathsf{c\_2Ewords\_2Eword\_\_msb}$ *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Definition 42 *We define* $\mathsf{c\_2Ebool\_2ELET}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Definition 43 *We define* $\mathsf{c\_2Ewords\_2Enzcv}$ *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b \in ($

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{25}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{26}$$

Definition 44 *We define* $\mathsf{c\_2Epair\_2EUNCURRY}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 45 *We define* $\mathsf{c\_2Ewords\_2Eword\_\_1comp}$ *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Definition 46 *We define* $\mathsf{c\_2Ewords\_2Eword\_\_ls}$ *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b$

Definition 47 *We define* $\mathsf{c\_2Ewords\_2Eword\_\_lo}$ *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)) \tag{27}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{29}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p))))) \tag{30}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))))) \tag{31}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{32}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m))))) \tag{33}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m))) \tag{34}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n)) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))))) \tag{35}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V2p))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1n)\ V2p)))))) \tag{36}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{37}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((V0m = V1n)\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m)))))) \tag{38}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{39}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(V0m = V1n))\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))\vee(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m)))))) \tag{40}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \tag{41}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n))\ V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)))))) \tag{42}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n))\ V2p))\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V2p)))))))) \tag{43}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n) = V2p)\Leftrightarrow((V0m = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p))\vee((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V2p)\ \mathsf{c\_2Enum\_2E0})))))))) \tag{44}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1k)\ V0n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n) = V1k)))) \tag{45}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EMOD\ \mathsf{c\_2Enum\_2E0})\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{46}$$

Assume the following.

$$(\forall V0r \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0r)\ V1n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0r)\ V1n) = \mathsf{c\_2Enum\_2E0})))) \tag{47}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow(((ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0n)\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ V0n) = \mathsf{c\_2Enum\_2E0})))) \tag{48}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V1y))\Rightarrow(((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0x)\ V1y) = V0x)\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0x)\ V1y)))))) \tag{49}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Enum\_2Enum.(\forall V2b \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1a)\ V2b)))\Leftrightarrow(\forall V3d \in ty\_2Enum\_2Enum.(((V2b = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1a)\ V3d))\Rightarrow(p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})))\wedge((V1a = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2b)\ V3d))\Rightarrow(p\ (ap\ V0P\ V3d))))))))) \tag{50}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V0n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0n))) \tag{51}$$

Assume the following.

$$(\forall V0r \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0r)\ V1n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EDIV\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0r))\ V1n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{52}$$

Assume the following.

$$(\forall V0h \in ty\_2Enum\_2Enum.(\forall V1l \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EBITS}\ V0h)\ V1l)\ V2n) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\ V2n)\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V1l)))\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0h))\ V1l))))))) \tag{53}$$

Assume the following.

$$True \tag{54}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{55}$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{58}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{59}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{60}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{61}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{62}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{65}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{67}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{68}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B))))) \tag{70}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{71}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{72}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\land((\forall V1n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\land((\forall V2n \in$$
$$ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap$$
$$(ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\land((\forall V4n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\land$$
$$((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\land((\forall V6n \in ty\_2Enum\_2Enum.$$
$$(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ ($$
$$ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V6n)\ V7m)))))\land((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\land((\forall V9n \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\land((\forall V10n \in$$
$$ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$V10n)\ V11m)))))\land((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$\mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$V12n))) = \mathsf{c\_2Enum\_2E0}))\land((\forall V13n \in ty\_2Enum\_2Enum.((ap$$
$$(ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\land((\forall V14n \in$$
$$ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) =$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\land$$
$$((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.$$
$$((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\land(((ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$\mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\land((\forall V17n \in ty\_2Enum\_2Enum.($$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\land(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) =$$
$$\mathsf{c\_2Enum\_2E0})\land((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\land((\forall V19n \in ty\_2Enum\_2Enum.$$
$$(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\land$$
$$((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\land((\forall V21n \in ty\_2Enum\_2Enum.$$
$$(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\land$$
$$((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\land((\forall V24n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V24n))))\land((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V25n)\ V26m)))))\land((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}$$
$$\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\land((\forall V28n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})$$
$$V28n))))\land((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V30m)\ V29n)))))\land((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}$$
$$\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\land((\forall V32n \in ty\_2Enum\_2Enum.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$
\begin{gathered}
(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = V0n) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (\\
ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (\\
ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (\\
ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
(((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)) \wedge (((ap \\
\mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
(ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge (\\
((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
(ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
(((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V0n)) = (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC} \\
(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO})) = (\\
ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (\\
ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V1m))) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ V1m)))) \wedge (((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))) = \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ V1m)))) \wedge ((ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))) = \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ (ap\ \mathsf{c\_2Enumeral\_2EiiSUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
V0n)\ V1m))))))))))))))))))))))
\end{gathered}
\tag{74}
$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\Leftrightarrow False)\wedge ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO})\Leftrightarrow False)\wedge(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO})\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m))\Leftrightarrow(V0n = V1m))\wedge(((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m))\Leftrightarrow(V0n = V1m)))))))))) \tag{75}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow (\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))) \tag{76}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m)))))))))) \tag{77}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{78}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) = (ap\ (ap\ V0f\ V1x)\ V2y))))) \tag{79}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{81}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{82}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{83}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{84}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{85}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg (p\ V1q))) \wedge (((p\ V0p) \vee (\neg (p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg (p\ V0p)))))))))) \tag{87}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg (p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg (p\ V1q)) \vee (\neg (p\ V0p))))))) \tag{88}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{89}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg ((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg (p\ V1q))))) \tag{90}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)) = (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))) \tag{91}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1y \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V2carry\_\_in \in 2.((ap\ (c\_2Ewords\_2Eadd\_\_with\_\_carry\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2))\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2)\ V1y)\ V2carry\_\_in))) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2)))\ (\lambda V3unsigned\_\_sum \in ty\_2Enum\_2Enum.(ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2)))\ (\lambda V4result \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ 2\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2)))\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ 2\ ((ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2))^{2}))\ (\lambda V5carry\_\_out \in 2.(\lambda V6overflow \in 2.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2))\ V4result)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ 2)\ V5carry\_\_out)\ V6overflow)))))\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V4result))\ V3unsigned\_\_sum))))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V1y)))\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V4result)))))))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V3unsigned\_\_sum))))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V1y)))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ V2carry\_\_in)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ \mathsf{c\_2Enum\_2E0})))))))) \tag{92}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))) \tag{93}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.((ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0n)) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))) \tag{94}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V0w)) = V0w)) \tag{95}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0m) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V1n)) \Leftrightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0m)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))) \quad (96)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\exists V1n \in ty\_2Enum\_2Enum.((V0w = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V1n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))) \quad (97)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0m))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V1n)) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))) \quad (98)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.((ap\ (\mathsf{c\_2Ewords\_2Eword\_\_1comp}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0n)) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))) \quad (99)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.((ap\ (\mathsf{c\_2Ewords\_2Eword\_\_2comp}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0n)) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))) \quad (100)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_1comp}\ A\_27a)\ V0w))) \Leftrightarrow (\neg(p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0w))))) \tag{101}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V2x \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ V0v)\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ V1w)\ V2x)) = (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ V0v)\ V1w))\ V2x))))) \tag{102}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((ap\ (\mathsf{c\_2Ewords\_2Eword\_\_2comp}\ A\_27a)\ V0w) = (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_add}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_1comp}\ A\_27a)\ V0w))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{103}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1b \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lo}\ A\_27a)\ V0a)\ V1b))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_ls}\ A\_27a)\ V1b)\ V0a))))) \tag{104}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1y \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((ap\ (c\_2Ewords\_2Eadd\_\_with\_\_carry\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2))\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ 2)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_1comp}\ A\_27a)\ V1y))\ \mathsf{c\_2Ebool\_2ET}))) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Epair\_2Eprod\ 2\ 2))\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_sub}\ A\_27a)\ V0x)\ V1y))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ 2\ 2)\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_ls}\ A\_27a)\ V1y)\ V0x))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0x))\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V1y))))\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_sub}\ A\_27a)\ V0x)\ V1y)))\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ V0x)))))))))$$