

thm_2Ewords_2EINT__MIN__11
(TMXswjKPK4LHoEVKQ6SZHrHru1wfuaSwAMn)

October 26, 2020

Let $ty_2Efc_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Ebit0\ A0) \quad (1)$$

Let $ty_2Efc_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Ebit1\ A0) \quad (2)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2E2T to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2E2T)$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (3)$$

Definition 4 We define c_2Ebool_2E2IN to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap\ V1f\ V0x)))$

Definition 5 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define c_2Ebool_2E21 to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a})))$

Definition 7 We define $c_2Ebool_2E5C_2E2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2$

Definition 8 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (4)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27b.nonempty\ A.\lambda 27b \Rightarrow c_2Epair_2EABS_prod\ A.\lambda 27a\ A.\lambda 27b \in ((ty_2Epair_2Eprod\ A.\lambda 27a\ A.\lambda 27b)^{(2^{A-27b})^{A-27a}}) \quad (5)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E$
 Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(6)

Definition 10 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 13 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_21 2)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum$$
(7)

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum})$$
(8)

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum})$$
(9)

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum})$$
(10)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega})$$
(11)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega})$$
(12)

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num m)$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge P x))$
 of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap (c_2Eprim_rec_2E_3C m n))$

Definition 29 We define $c_2\text{Earithmetic_2EZERO}$ to be $c_2\text{Enum_2E0}$.

Definition 30 We define $c_2\text{Earithmetic_2EBIT2}$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2\text{Earithmetic_2E2A}$

Definition 31 We define $c_2\text{Earithmetic_2ENUMERAL}$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2\text{Earithmetic_2EEXP} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_2EEXP} \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Let $c_2\text{Ebool_2Ethe_value} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2\text{Ebool_2Ethe_value } A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (22)$$

Let $c_2\text{Ewords_2EINT_MIN} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2\text{Ewords_2EINT_MIN } A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2\text{Earithmetic_2E_2A} (\\ & ap c_2\text{Earithmetic_2ENUMERAL} (ap c_2\text{Earithmetic_2EBIT2 } c_2\text{Earithmetic_2EZERO)))) \\ & V0n) = (ap (ap c_2\text{Earithmetic_2E_2B } V0n) V0n))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap (ap (ap (c_2\text{Ebool_2ECOND } A_27a) c_2\text{Ebool_2ET}) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2\text{Ebool_2ECOND } A_27a) c_2\text{Ebool_2EF} \\ & V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow ((ap (c_2\text{Efc_2Edimindex} (ty_2Efc_2Ebit0 \\ & A_27a)) (c_2\text{Ebool_2Ethe_value} (ty_2Efc_2Ebit0 A_27a))) = (\\ & ap (ap (ap (c_2\text{Ebool_2ECOND } ty_2Enum_2Enum) (ap (c_2\text{Epred_set_2EFINITE} \\ & A_27a) (c_2\text{Epred_set_2EUNIV } A_27a))) (ap (ap c_2\text{Earithmetic_2E_2A} \\ & (ap c_2\text{Earithmetic_2ENUMERAL} (ap c_2\text{Earithmetic_2EBIT2 } c_2\text{Earithmetic_2EZERO)))) \\ & (ap (c_2\text{Efc_2Edimindex} A_27a) (c_2\text{Ebool_2Ethe_value} A_27a))) \\ & (ap c_2\text{Earithmetic_2ENUMERAL} (ap c_2\text{Earithmetic_2EBIT1 } c_2\text{Earithmetic_2EZERO)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((p\ (ap\ (c.2Epred_set.2EFINITE \\ & (ty.2EfcP.2Ebit0\ A.27a))\ (c.2Epred_set.2EUNIV\ (ty.2EfcP.2Ebit0 \\ & A.27a)))) \Leftrightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (c.2Epred_set.2EUNIV \\ & A.27a)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((ap\ (c.2EfcP.2Edimindex\ (ty.2EfcP.2Ebit1 \\ & A.27a))\ (c.2Ebool.2Ethe_value\ (ty.2EfcP.2Ebit1\ A.27a))) = (\\ & ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ ty.2Enum.2Enum)\ (ap\ (c.2Epred_set.2EFINITE \\ & A.27a)\ (c.2Epred_set.2EUNIV\ A.27a)))\ (ap\ (ap\ c.2Earithmetic.2E.2B \\ & (ap\ (ap\ c.2Earithmetic.2E.2A\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap \\ & c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))))\ (ap\ (c.2EfcP.2Edimindex \\ & A.27a)\ (c.2Ebool.2Ethe_value\ A.27a))))\ (ap\ c.2Earithmetic.2ENUMERAL \\ & (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))\ (ap\ c.2Earithmetic.2ENUMERAL \\ & (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((p\ (ap\ (c.2Epred_set.2EFINITE \\ & (ty.2EfcP.2Ebit1\ A.27a))\ (c.2Epred_set.2EUNIV\ (ty.2EfcP.2Ebit1 \\ & A.27a)))) \Leftrightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (c.2Epred_set.2EUNIV \\ & A.27a)))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} ((ap\ (c.2EfcP.2Edimindex\ ty.2Eone.2Eone)\ (c.2Ebool.2Ethe_value \\ ty.2Eone.2Eone)) = (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\ c.2Earithmetic.2EZERO))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (p\ (ap\ (c.2Epred_set.2EFINITE\ ty.2Eone.2Eone)\ (c.2Epred_set.2EUNIV \\ ty.2Eone.2Eone))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (((ap\ c.2Enum.2ESUC\ c.2Earithmetic.2EZERO) = (ap\ c.2Earithmetic.2EBIT1 \\ c.2Earithmetic.2EZERO)) \wedge ((\forall V0n \in ty.2Enum.2Enum.((ap \\ c.2Enum.2ESUC\ (ap\ c.2Earithmetic.2EBIT1\ V0n)) = (ap\ c.2Earithmetic.2EBIT2 \\ V0n))) \wedge (\forall V1n \in ty.2Enum.2Enum.((ap\ c.2Enum.2ESUC\ (ap\ c.2Earithmetic.2EBIT2 \\ V1n)) = (ap\ c.2Earithmetic.2EBIT1\ (ap\ c.2Enum.2ESUC\ V1n)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.(\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) (ap c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& (ap c_2Earithmic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge \\
& (((ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)) = \\
& c_2Earithmic_2EZERO) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\
& c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 (ap c_2Earithmic_2EBIT1 \\
& V0n))) = (ap c_2Earithmic_2EBIT2 (ap c_2Eprim_rec_2EPRE (ap \\
& c_2Earithmic_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 (ap c_2Earithmic_2EBIT2 \\
& V1n))) = (ap c_2Earithmic_2EBIT2 (ap c_2Earithmic_2EBIT1 \\
& V1n)))) \wedge ((\forall V2n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE \\
& (ap c_2Earithmic_2EBIT2 V2n)) = (ap c_2Earithmic_2EBIT1 V2n))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1b \in 2. (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3m \in ty_2Enum_2Enum. (((ap (ap (ap c_2Enumeral_2EiSUB \\
& V1b) c_2Earithmetic_2EZERO) V0x) = c_2Earithmetic_2EZERO) \wedge (\\
& ((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) c_2Earithmetic_2EZERO) = \\
V2n) \wedge (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
V2n)) c_2Earithmetic_2EZERO) = (ap c_2Enumeral_2EiDUB V2n)) \wedge \\
(((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
(ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
(ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
(ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
(ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
V2n)) c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 V2n)) \wedge \\
(((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
(ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
(ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
(ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge ((ap \\
(ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
(ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))))))))))))))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V0n) \\
V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\
V1m) V0n)) (ap c_2Earithmetic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB \\
c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0)))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
(\forall V0n \in ty_2Enum_2Enum.(((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
(((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap\ c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO))))))
\end{aligned}
\tag{41}$$

Assume the following.

$$\begin{aligned}
(\forall V0acc \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\
(((ap\ (ap\ c_2Enumeral_2Etepx_help\ c_2Earithmetic_2EZERO)\ V0acc) = (ap\ c_2Earithmetic_2EBIT2\ V0acc)) \wedge (((ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Earithmetic_2EBIT1\ V1n))\ V0acc) = (ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmetic_2EBIT1\ V1n))\ (ap\ c_2Earithmetic_2EBIT1\ V0acc))) \wedge ((ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Earithmetic_2EBIT2\ V1n))\ V0acc) = (ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Earithmetic_2EBIT1\ V1n))\ (ap\ c_2Earithmetic_2EBIT1\ V0acc))))))))))
\end{aligned}
\tag{42}$$

Assume the following.

$$\begin{aligned}
(\forall V0n \in ty_2Enum_2Enum.(((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ c_2Enum_2E0) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) \wedge (((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ c_2Earithmetic_2EZERO))) \wedge ((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ V0n))) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ c_2Earithmetic_2EZERO))))))))))
\end{aligned}
\tag{43}$$

Assume the following.

$$\begin{aligned}
(\forall V0i \in ty_2Enum_2Enum.((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2ENUMERAL\ V0i)) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Enumeral_2EiDUB\ V0i))))
\end{aligned}
\tag{44}$$

Assume the following.

$$\begin{aligned}
\forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Ewords_2EINT_MIN\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)) = (ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2Efc_2Edimindex\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))
\end{aligned}
\tag{45}$$

Theorem 1

$$\begin{aligned} & ((ap (c_Ewords_EINT_MIN (ty_Efc_Ebit1 (ty_Efc_Ebit1 \\ & \quad (ty_Efc_Ebit0 ty_Eone_Eone)))) (c_Ebool_Ethe_value \\ & (ty_Efc_Ebit1 (ty_Efc_Ebit1 (ty_Efc_Ebit0 ty_Eone_Eone)))))) = \\ & (ap c_Earithmetic_ENUMERAL (ap c_Earithmetic_EBIT2 (ap c_Earithmetic_EBIT1 \\ & \quad (ap c_Earithmetic_EBIT1 (ap c_Earithmetic_EBIT1 (ap c_Earithmetic_EBIT1 \\ & \quad (ap c_Earithmetic_EBIT1 (ap c_Earithmetic_EBIT1 (ap c_Earithmetic_EBIT1 \\ & \quad (ap c_Earithmetic_EBIT1 (ap c_Earithmetic_EBIT1 c_Earithmetic_EZERO))))))))))))) \end{aligned}$$