

thm\_2Ewords\_2EINT\_\_MIN\_\_24  
(TMbthHRYyGKx1VC6qFY3oJwDv3mBpp8Wk3p)

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Let  $ty\_2Efc\_2Ebit0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efc\_2Ebit0\ A0) \quad (1)$$

Let  $ty\_2Efc\_2Ebit1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efc\_2Ebit1\ A0) \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E2T$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E2T)$ .

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (3)$$

**Definition 4** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 5** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A\_27a})))$

**Definition 7** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2$

**Definition 8** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in 2$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (4)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (5)$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
(6)

**Definition 10** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define  $c\_2Ebool\_2E\_21 2$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 12** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_21 2)$ .

**Definition 13** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum$$
(7)

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum})$$
(8)

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum})$$
(9)

**Definition 14** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum})$$
(10)

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega})$$
(11)

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega})$$
(12)

**Definition 15** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num V0m)$ .

**Definition 16** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x) of type \iota \Rightarrow \iota)$ .

**Definition 17** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) V0P)))$ .

**Definition 18** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap c\_2Eprim\_rec\_2E\_3C V0m V1n)$ .



**Definition 29** We define  $c\_2\text{Earithmic\_2EZERO}$  to be  $c\_2\text{Enum\_2E0}$ .

**Definition 30** We define  $c\_2\text{Earithmic\_2EBIT2}$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2\text{Earithmic\_2E2A}$

**Definition 31** We define  $c\_2\text{Earithmic\_2ENUMERAL}$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2\text{Earithmic\_2EEXP} : \iota$  be given. Assume the following.

$$c\_2\text{Earithmic\_2EEXP} \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (21)$$

Let  $c\_2\text{Ebool\_2Ethe\_value} : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2\text{Ebool\_2Ethe\_value } A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (22)$$

Let  $c\_2\text{Ewords\_2EINT\_MIN} : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2\text{Ewords\_2EINT\_MIN } A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (23)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2\text{Earithmic\_2E2A} (ap c\_2\text{Earithmic\_2ENUMERAL} (ap c\_2\text{Earithmic\_2EBIT2 } c\_2\text{Earithmic\_2EZERO)))) V0n) = (ap (ap c\_2\text{Earithmic\_2E2B } V0n) V0n))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap (ap (ap (c\_2\text{Ebool\_2ECOND } A\_27a) c\_2\text{Ebool\_2ET}) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2\text{Ebool\_2ECOND } A\_27a) c\_2\text{Ebool\_2EF}) V0t1) V1t2) = V1t2)))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\text{ap } (c\_2Efc\_2Edimindex \ (ty\_2Efc\_2Ebit0 \ A_{27a})) \ (c\_2Ebool\_2Ethe\_value \ (ty\_2Efc\_2Ebit0 \ A_{27a}))) = ( \\
& \text{ap } (\text{ap } (\text{ap } (c\_2Ebool\_2ECOND \ ty\_2Enum\_2Enum) \ (\text{ap } (c\_2Epred\_set\_2EFINITE \ A_{27a})) \ (c\_2Epred\_set\_2EUNIV \ A_{27a}))) \ (\text{ap } (\text{ap } c\_2Earithmetic\_2E\_2A \\
& (\text{ap } c\_2Earithmetic\_2ENUMERAL \ (\text{ap } c\_2Earithmetic\_2EBIT2 \ c\_2Earithmetic\_2EZERO)))) \\
& (\text{ap } (c\_2Efc\_2Edimindex \ A_{27a})) \ (c\_2Ebool\_2Ethe\_value \ A_{27a}))) \\
& (\text{ap } c\_2Earithmetic\_2ENUMERAL \ (\text{ap } c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((p \ (\text{ap } (c\_2Epred\_set\_2EFINITE \\
& (ty\_2Efc\_2Ebit0 \ A_{27a})) \ (c\_2Epred\_set\_2EUNIV \ (ty\_2Efc\_2Ebit0 \\
& A_{27a})))) \Leftrightarrow (p \ (\text{ap } (c\_2Epred\_set\_2EFINITE \ A_{27a})) \ (c\_2Epred\_set\_2EUNIV \\
& A_{27a}))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\text{ap } (c\_2Efc\_2Edimindex \ (ty\_2Efc\_2Ebit1 \ A_{27a})) \ (c\_2Ebool\_2Ethe\_value \ (ty\_2Efc\_2Ebit1 \ A_{27a}))) = ( \\
& \text{ap } (\text{ap } (\text{ap } (c\_2Ebool\_2ECOND \ ty\_2Enum\_2Enum) \ (\text{ap } (c\_2Epred\_set\_2EFINITE \ A_{27a})) \ (c\_2Epred\_set\_2EUNIV \ A_{27a}))) \ (\text{ap } (\text{ap } c\_2Earithmetic\_2E\_2B \\
& (\text{ap } (\text{ap } c\_2Earithmetic\_2E\_2A \ (\text{ap } c\_2Earithmetic\_2ENUMERAL \ (\text{ap } \\
& c\_2Earithmetic\_2EBIT2 \ c\_2Earithmetic\_2EZERO)))) \ (\text{ap } (c\_2Efc\_2Edimindex \\
& A_{27a})) \ (c\_2Ebool\_2Ethe\_value \ A_{27a})))) \ (\text{ap } c\_2Earithmetic\_2ENUMERAL \\
& (\text{ap } c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO)))) \ (\text{ap } c\_2Earithmetic\_2ENUMERAL \\
& (\text{ap } c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((p \ (\text{ap } (c\_2Epred\_set\_2EFINITE \\
& (ty\_2Efc\_2Ebit1 \ A_{27a})) \ (c\_2Epred\_set\_2EUNIV \ (ty\_2Efc\_2Ebit1 \\
& A_{27a})))) \Leftrightarrow (p \ (\text{ap } (c\_2Epred\_set\_2EFINITE \ A_{27a})) \ (c\_2Epred\_set\_2EUNIV \\
& A_{27a}))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& ((\text{ap } (c\_2Efc\_2Edimindex \ ty\_2Eone\_2Eone) \ (c\_2Ebool\_2Ethe\_value \\
& ty\_2Eone\_2Eone)) = (\text{ap } c\_2Earithmetic\_2ENUMERAL \ (\text{ap } c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (p \ (\text{ap } (c\_2Epred\_set\_2EFINITE \ ty\_2Eone\_2Eone) \ (c\_2Epred\_set\_2EUNIV \\
& ty\_2Eone\_2Eone))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
 &(((ap\ c\_2Enum\_2ESUC\ c\_2Earithmetic\_2EZERO) = (ap\ c\_2Earithmetic\_2EBIT1 \\
 &\quad c\_2Earithmetic\_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap \\
 &\quad c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2 \\
 &\quad V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap\ c\_2Enum\_2ESUC\ (ap\ c\_2Earithmetic\_2EBIT2 \\
 &\quad V1n)) = (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enum\_2ESUC\ V1n))))))
 \end{aligned}
 \tag{35}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))
\end{aligned}$$

(37)



Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) (ap c\_2Earithmic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\
& (ap c\_2Earithmic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Eprim\_rec\_2EPRE c\_2Earithmic\_2EZERO) = c\_2Earithmic\_2EZERO) \wedge \\
& (((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)) = \\
& c\_2Earithmic\_2EZERO) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap \\
& c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 (ap c\_2Earithmic\_2EBIT1 \\
& V0n))) = (ap c\_2Earithmic\_2EBIT2 (ap c\_2Eprim\_rec\_2EPRE (ap \\
& c\_2Earithmic\_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmic\_2EBIT1 (ap c\_2Earithmic\_2EBIT2 \\
& V1n))) = (ap c\_2Earithmic\_2EBIT2 (ap c\_2Earithmic\_2EBIT1 \\
& V1n)))) \wedge ((\forall V2n \in ty\_2Enum\_2Enum. ((ap c\_2Eprim\_rec\_2EPRE \\
& (ap c\_2Earithmic\_2EBIT2 V2n)) = (ap c\_2Earithmic\_2EBIT1 V2n))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1b \in 2. (\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3m \in ty\_2Enum\_2Enum. (((ap (ap (ap c\_2Enumeral\_2EiSUB \\
& V1b) c\_2Earithmic\_2EZERO) V0x) = c\_2Earithmic\_2EZERO) \wedge ( \\
& ((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) c\_2Earithmic\_2EZERO) = \\
V2n) \wedge (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT1 \\
V2n)) c\_2Earithmic\_2EZERO) = (ap c\_2Enumeral\_2EiDUB V2n)) \wedge \\
(((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmic\_2EBIT1 \\
V2n)) (ap c\_2Earithmic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT1 \\
V2n)) (ap c\_2Earithmic\_2EBIT1 V3m)) = (ap c\_2Earithmic\_2EBIT1 \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmic\_2EBIT1 \\
V2n)) (ap c\_2Earithmic\_2EBIT2 V3m)) = (ap c\_2Earithmic\_2EBIT1 \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT1 \\
V2n)) (ap c\_2Earithmic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT2 \\
V2n)) c\_2Earithmic\_2EZERO) = (ap c\_2Earithmic\_2EBIT1 V2n)) \wedge \\
(((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmic\_2EBIT2 \\
V2n)) (ap c\_2Earithmic\_2EBIT1 V3m)) = (ap c\_2Earithmic\_2EBIT1 \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT2 \\
V2n)) (ap c\_2Earithmic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmic\_2EBIT2 \\
V2n)) (ap c\_2Earithmic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge ((ap \\
(ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmic\_2EBIT2 \\
V2n)) (ap c\_2Earithmic\_2EBIT2 V3m)) = (ap c\_2Earithmic\_2EBIT1 \\
(ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))))))))))))))))) \\
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& (ap c\_2Earithmic\_2ENUMERAL (ap (ap c\_2Earithmic\_2E\_2D V0n) \\
V1m)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V1m) V0n)) (ap c\_2Earithmic\_2ENUMERAL (ap (ap (ap c\_2Enumeral\_2EiSUB \\
c\_2Ebool\_2ET) V0n) V1m))) c\_2Enum\_2E0)))) \\
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
(\forall V0n \in ty\_2Enum\_2Enum.(((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiDUB\ V0n))) \wedge \\
(((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) \wedge ((ap\ c\_2Enumeral\_2EiDUB\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO))))))
\end{aligned}
\tag{42}$$

Assume the following.

$$\begin{aligned}
(\forall V0acc \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\
(((ap\ (ap\ c\_2Enumeral\_2Etepx\_help\ c\_2Earithmetic\_2EZERO)\ V0acc) = (ap\ c\_2Earithmetic\_2EBIT2\ V0acc)) \wedge (((ap\ (ap\ c\_2Enumeral\_2Etepx\_help\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n))\ V0acc) = (ap\ (ap\ c\_2Enumeral\_2Etepx\_help\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V0acc))) \wedge ((ap\ (ap\ c\_2Enumeral\_2Etepx\_help\ (ap\ c\_2Earithmetic\_2EBIT2\ V1n))\ V0acc) = (ap\ (ap\ c\_2Enumeral\_2Etepx\_help\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V0acc))))))))))
\end{aligned}
\tag{43}$$

Assume the following.

$$\begin{aligned}
(\forall V0n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ c\_2Enum\_2E0) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) \wedge (((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ (ap\ c\_2Enumeral\_2Etepx\_help\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ c\_2Earithmetic\_2EZERO))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n))) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ (ap\ c\_2Enumeral\_2Etepx\_help\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ c\_2Earithmetic\_2EZERO))))))))))
\end{aligned}
\tag{44}$$

Assume the following.

$$\begin{aligned}
(\forall V0i \in ty\_2Enum\_2Enum.((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2ENUMERAL\ V0i)) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Enumeral\_2EiDUB\ V0i))))
\end{aligned}
\tag{45}$$

Assume the following.

$$\begin{aligned}
\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Ewords\_2EINT\_MIN\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a)) = (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efc\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a)))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))
\end{aligned}
\tag{46}$$

