

thm_2Ewords_2EINT__MIN__8
(TMX1Y2QnYn5d4vembvsAaFku3ZQPAGcbA1P)

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Let $ty_2Efc_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Ebit0\ A0) \quad (1)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2E2T to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2E2T)$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (2)$$

Definition 4 We define c_2Ebool_2EIN to be $\lambda A.\lambda 27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap\ V1f\ V0x)))$

Definition 5 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define c_2Ebool_2E21 to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ P)\ V0P))$

Definition 7 We define $c_2Ebool_2E5C_2E2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t)))$

Definition 8 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27b.nonempty\ A.\lambda 27b \Rightarrow c_2Epair_2EABS_prod\ A.\lambda 27a.\lambda 27b \in ((ty_2Epair_2Eprod\ A.\lambda 27a.\lambda 27b)^{(2^{A-27b})^{A-27a}}) \quad (4)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E$
 Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}})$$
(5)

Definition 10 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 13 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_21\ 2)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum$$
(6)

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum})$$
(7)

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum})$$
(8)

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2)$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(9)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{omega})$$
(10)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega})$$
(11)

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap\ P\ x)) \mathbf{then} (the (\lambda x. x \in A \wedge P\ x))$
 of type $\iota \Rightarrow \iota$.

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap (c_2Emin_2E_40\ A_27a))))$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ c_2Eprim_rec_2E_3C\ m\ n)$

Definition 19 We define $c_2Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Earithmic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 22 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2) \quad (14)$$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (15)$$

Definition 24 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 25 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Enumeral_2Eteexp_help : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eteexp_help \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Definition 26 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmic_2E_2A$

Definition 27 We define $c_2Earithmic_2E_2BIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmic_2E_2A$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (17)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2EfcP_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (18)$$

Let $c_2Earithmic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 28 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 29 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E2A$

Definition 30 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (21)$$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EINT_MIN A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (22)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V0n) = (ap (ap c_2Earithmetic_2E2B V0n) V0n))) \quad (23)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\text{ap } (c_2Efc_2Edimindex \ (ty_2Efc_2Ebit0 \\
& \quad A_{27a})) \ (c_2Ebool_2Ethe_value \ (ty_2Efc_2Ebit0 \ A_{27a}))) = (\\
& \text{ap } (\text{ap } (\text{ap } (c_2Ebool_2ECOND \ ty_2Enum_2Enum) \ (\text{ap } (c_2Epred_set_2EFINITE \\
& \quad A_{27a}) \ (c_2Epred_set_2EUNIV \ A_{27a}))) \ (\text{ap } (\text{ap } c_2Earithmetic_2E_2A \\
& \quad (\text{ap } c_2Earithmetic_2ENUMERAL \ (\text{ap } c_2Earithmetic_2EBIT2 \ c_2Earithmetic_2EZERO)))) \\
& \quad (\text{ap } (c_2Efc_2Edimindex \ A_{27a}) \ (c_2Ebool_2Ethe_value \ A_{27a}))) \\
& \quad (\text{ap } c_2Earithmetic_2ENUMERAL \ (\text{ap } c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((p \ (\text{ap } (c_2Epred_set_2EFINITE \\
& \quad (ty_2Efc_2Ebit0 \ A_{27a})) \ (c_2Epred_set_2EUNIV \ (ty_2Efc_2Ebit0 \\
& \quad A_{27a})))) \Leftrightarrow (p \ (\text{ap } (c_2Epred_set_2EFINITE \ A_{27a}) \ (c_2Epred_set_2EUNIV \\
& \quad A_{27a}))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& ((\text{ap } (c_2Efc_2Edimindex \ ty_2Eone_2Eone) \ (c_2Ebool_2Ethe_value \\
& \quad ty_2Eone_2Eone)) = (\text{ap } c_2Earithmetic_2ENUMERAL \ (\text{ap } c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (p \ (\text{ap } (c_2Epred_set_2EFINITE \ ty_2Eone_2Eone) \ (c_2Epred_set_2EUNIV \\
& \quad ty_2Eone_2Eone)))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) (ap c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& (ap c_2Earithmic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge \\
& (((ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)) = \\
& c_2Earithmic_2EZERO) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\
& c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 (ap c_2Earithmic_2EBIT1 \\
& V0n))) = (ap c_2Earithmic_2EBIT2 (ap c_2Eprim_rec_2EPRE (ap \\
& c_2Earithmic_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 (ap c_2Earithmic_2EBIT2 \\
& V1n))) = (ap c_2Earithmic_2EBIT2 (ap c_2Earithmic_2EBIT1 \\
& V1n)))) \wedge ((\forall V2n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE \\
& (ap c_2Earithmic_2EBIT2 V2n)) = (ap c_2Earithmic_2EBIT1 V2n))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1b \in 2. (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3m \in ty_2Enum_2Enum. (((ap (ap (ap c_2Enumeral_2EiSUB \\
& V1b) c_2Earithmetic_2EZERO) V0x) = c_2Earithmetic_2EZERO) \wedge (\\
& ((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) c_2Earithmetic_2EZERO) = \\
& V2n) \wedge (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) c_2Earithmetic_2EZERO) = (ap c_2Enumeral_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge ((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))))))))))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V0n) \\
& V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n)) (ap c_2Earithmetic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB \\
& c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0)))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
(\forall V0n \in ty_2Enum_2Enum.(((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
(((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap\ c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO))))))
\end{aligned}
\tag{37}$$

Assume the following.

$$\begin{aligned}
(\forall V0acc \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. \\
(((ap\ (ap\ c_2Enumeral_2Etepx_help\ c_2Earithmetic_2EZERO)\ V0acc) = (ap\ c_2Earithmetic_2EBIT2\ V0acc)) \wedge (((ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Earithmetic_2EBIT1\ V1n))\ V0acc) = (ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmetic_2EBIT1\ V1n))\ (ap\ c_2Earithmetic_2EBIT1\ V0acc))) \wedge ((ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Earithmetic_2EBIT2\ V1n))\ V0acc) = (ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Earithmetic_2EBIT1\ V1n))\ (ap\ c_2Earithmetic_2EBIT1\ V0acc))))))))))
\end{aligned}
\tag{38}$$

Assume the following.

$$\begin{aligned}
(\forall V0n \in ty_2Enum_2Enum.(((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ c_2Enum_2E0) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))) \wedge (((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ c_2Earithmetic_2EZERO))) \wedge ((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ V0n))) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ (ap\ c_2Enumeral_2Etepx_help\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ c_2Earithmetic_2EZERO))))))))))
\end{aligned}
\tag{39}$$

Assume the following.

$$\begin{aligned}
(\forall V0i \in ty_2Enum_2Enum.((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2ENUMERAL\ V0i)) = (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Enumeral_2EiDUB\ V0i))))
\end{aligned}
\tag{40}$$

Assume the following.

$$\begin{aligned}
\forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Ewords_2EINT_MIN\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)) = (ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2Efcpx_2Edimindex\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))
\end{aligned}
\tag{41}$$

Theorem 1

$$\begin{aligned} & ((ap (c_2Ewords_2EINT_MIN (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 \\ & \quad (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) (c_2Ebool_2Ethe_value \\ & \quad (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) = \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\ & \quad (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\ & \quad (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))))))) \end{aligned}$$