

thm_2Ewords_2ELSR__BITWISE
(TMJ2WQbkhk2RLgU1rbT5D4kCqtiXj7dJD4d6)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Efcf_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efcf_2Efinite_image A0) \quad (1)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (2)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (3)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (4)$$

Let $c_2Efcf_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcf_2Edimindex A_27a \in (ty_2Eenum_2Eenum^{(ty_2Ebool_2Eitself A_27a)}) \quad (5)$$

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 13 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (9)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})(ty_2Efcp_2Ecart A_27a A_27b)) \quad (10)$$

Definition 14 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a$

Definition 15 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 16 We define $c_2Ewords_2Eword_and$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efcp_2Ecart 2 A_27a).\lambda V1$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Definition 18 We define $c_2Ewords_2Eword_or$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efcp_2Ecart 2 A_27a).\lambda V1$

Definition 19 We define $c_2Ewords_2Eword_xor$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efcp_2Ecart 2 A_27a).\lambda V1$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \quad (11)$$

Definition 20 We define $c_2Ewords_2Eword_lsr$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow \\
& ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27}))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\\
& \forall V0x \in (ty_2Efc_2Ecart A_{27a} A_{27b}).(\forall V1y \in (ty_2Efc_2Ecart \\
& A_{27a} A_{27b}).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_2Enum_2Enum.((p (ap \\
& (ap c_2Eprim_rec_2E_3C V2i) (ap (c_2Efc_2Edimindex A_{27b}) (\\
& c_2Ebool_2Ethe_value A_{27b})))) \Rightarrow ((ap (ap (c_2Efc_2Efc_index \\
& A_{27a} A_{27b}) V0x) V2i) = (ap (ap (c_2Efc_2Efc_index A_{27a} A_{27b}) \\
& V1y) V2i))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow (\\
& \forall V0g \in (A_{27a}^{ty_2Enum_2Enum}).(\forall V1i \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C V1i) (ap (c_2Efc_2Edimindex A_{27b}) \\
& (c_2Ebool_2Ethe_value A_{27b})))) \Rightarrow ((ap (ap (c_2Efc_2Efc_index \\
& A_{27a} A_{27b}) (ap (c_2Efc_2EFCP A_{27a} A_{27b}) V0g)) V1i) = (ap V0g \\
& V1i))))))
\end{aligned} \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \tag{26}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow (\neg(p V0A)) \Rightarrow False)) \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow (\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (32) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0n \in ty_2Enum_2Enum. \\ & (\forall V1v \in (ty_2Efc_2Ecart\ 2\ A.27a).(\forall V2w \in (ty_2Efc_2Ecart \\ & 2\ A.27a).((ap\ (ap\ (c_2Ewords_2Eword_and\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_lstr \\ & A.27a)\ V2w)\ V0n))\ (ap\ (ap\ (c_2Ewords_2Eword_lstr\ A.27a)\ V1v)\ V0n)) = \\ & (ap\ (ap\ (c_2Ewords_2Eword_lstr\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_and \\ & A.27a)\ V2w)\ V1v))\ V0n)))) \wedge ((\forall V3n \in ty_2Enum_2Enum.(\forall V4v \in \\ & (ty_2Efc_2Ecart\ 2\ A.27a).(\forall V5w \in (ty_2Efc_2Ecart\ 2 \\ & A.27a).((ap\ (ap\ (c_2Ewords_2Eword_or\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_lstr \\ & A.27a)\ V5w)\ V3n))\ (ap\ (ap\ (c_2Ewords_2Eword_lstr\ A.27a)\ V4v)\ V3n)) = \\ & (ap\ (ap\ (c_2Ewords_2Eword_lstr\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_or \\ & A.27a)\ V5w)\ V4v))\ V3n)))) \wedge ((\forall V6n \in ty_2Enum_2Enum.(\forall V7v \in \\ & (ty_2Efc_2Ecart\ 2\ A.27a).(\forall V8w \in (ty_2Efc_2Ecart\ 2 \\ & A.27a).((ap\ (ap\ (c_2Ewords_2Eword_xor\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_lstr \\ & A.27a)\ V8w)\ V6n))\ (ap\ (ap\ (c_2Ewords_2Eword_lstr\ A.27a)\ V7v)\ V6n)) = \\ & (ap\ (ap\ (c_2Ewords_2Eword_lstr\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_xor \\ & A.27a)\ V8w)\ V7v))\ V6n)))))) \end{aligned}$$