

thm_2Ewords_2EROL__MOD (TM- cuWwXqR7dWQ6xWYMq5VJbE17eR6Emm269)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty \ ty_2Eenum_2Eenum \tag{1}$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Eenum_2Eenum)^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum} \tag{2}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Eenum_2Eenum)^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum} \tag{3}$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty_2Ebool_2Eitself \ A0) \tag{4}$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow c_2Ebool_2Ethe_value \ A_27a \in (ty_2Ebool_2Eitself \ A_27a) \tag{5}$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2EfcP_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (6)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Efinite_image\ A0) \quad (8)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (V0m))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0P)))$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap\ V0m\ (c_2Eprim_rec_2E_3C\ V1n))$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E_2F_5C\ A_27a)\ V0P)))$

Definition 13 We define $c_2EfcP_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E_40\ A_27a)\ (ty_2Enum_2Enum^{ty_2Enum_2Enum}))$

Let $ty_2EfcP_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2EfcP_2Ecart\ A0\ A1) \quad (12)$$

Let $c_2EfcP_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EfcP_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2EfcP_2Efinite_image\ A_27b)})^{(ty_2EfcP_2Ecart\ A_27a\ A_27b)}) \quad (13)$$

Definition 14 We define $c_2EfcP_2EfcP_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2EfcP_2Ecart\ A_27a\ A_27b).(ap\ V0x\ (c_2EfcP_2EfcP_index\ A_27a\ A_27b))$

Definition 15 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 16 We define $c_2Ewords_2Eword_ror$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1$

Definition 17 We define $c_2Ewords_2Eword_rol$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{14}$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ & c_2Enum_2E0)\ V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2EMOD \\ & (ap\ (ap\ c_2Earithmetic_2EMOD\ V1k)\ V0n))\ V0n) = (ap\ (ap\ c_2Earithmetic_2EMOD \\ & V1k)\ V0n)))))) \end{aligned} \tag{15}$$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{17}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0) \\ & (ap\ (c_2Efc_2Edimindex\ A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))) \end{aligned} \tag{19}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Efc_2Ecart \\ & 2\ A_27a).(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ewords_2Eword_rol \\ & A_27a)\ V0w)\ (ap\ (ap\ c_2Earithmetic_2EMOD\ V1n)\ (ap\ (c_2Efc_2Edimindex \\ & A_27a)\ (c_2Ebool_2Ethe_value\ A_27a)))))) = (ap\ (ap\ (c_2Ewords_2Eword_rol \\ & A_27a)\ V0w)\ V1n)))) \end{aligned}$$