

thm_2Ewords_2EWORD__2COMP__LSL
(TMar5b2X4Erg5o257YULosz9yhgucuvdpJ6)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Efc_2E_finite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efc_2E_finite_image A0) \quad (1)$$

Let $ty_2Ebool_2E_itself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2E_itself A0) \quad (2)$$

Let $c_2Ebool_2E_the_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2E_the_value A_27a \in (ty_2Ebool_2E_itself A_27a) \quad (3)$$

Let $ty_2Eenum_2E_enum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2E_enum \quad (4)$$

Let $c_2Efc_2E_dimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efc_2E_dimindex A_27a \in (ty_2Eenum_2E_enum^{(ty_2Ebool_2E_itself A_27a)}) \quad (5)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 17 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 18 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 20 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebo$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 21 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap (ap c$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)})^{ty_2Enum_2Enum} \quad (15)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Definition 22 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (17)$$

Definition 23 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (18)$$

Definition 24 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 26 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 27 We define c_2EfcP_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 28 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2EfcP_2EFC$

Definition 29 We define $c_Ewords_Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 30 We define $c_Ewords_Eword_mul$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V$

Definition 31 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 32 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 33 We define $c_Ewords_Eword_lsl$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).(\forall V1w \in (ty_2Efc_2Ecart\ 2\ A_27a).((ap\ (c_2Ewords_Eword_2comp \\ & A_27a)\ (ap\ (ap\ (c_2Ewords_Eword_mul\ A_27a)\ V0v)\ V1w))) = (ap\ (ap \\ & (c_2Ewords_Eword_mul\ A_27a)\ V0v)\ (ap\ (c_2Ewords_Eword_2comp\ A_27a)\ V1w)))))) \quad (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ewords_Eword_lsl \\ & A_27a)\ V0a)\ V1n) = (ap\ (ap\ (c_2Ewords_Eword_mul\ A_27a)\ (ap\ (c_2Ewords_2En2w \\ & A_27a)\ (ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))\ V1n)))\ V0a)))))) \quad (23) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\forall V1a \in (ty_2Efc_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_Eword_lsl \\ & A_27a)\ (ap\ (c_2Ewords_Eword_2comp\ A_27a)\ V1a))\ V0n) = (ap\ (c_2Ewords_Eword_2comp \\ & A_27a)\ (ap\ (ap\ (c_2Ewords_Eword_lsl\ A_27a)\ V1a)\ V0n)))))) \end{aligned}$$