thm_2Ewords_2EWORD__ADD__0 (TMW2oaaEU4xK5gBTaGXaWzDQwtPXE8fjVuS)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (3)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27})$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \qquad (4)$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \qquad (5)$$

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Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (\\ ty\_2Ebool\_2Eitself\ A\_27a) \tag{6}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{7}$$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{9}$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ $c_2Ebool_2E_2F_5C

Definition 14 *We define* c_2Efcp_2Efinite_index *to be* $\lambda A\_27a : \iota.(ap\ ($c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\\ A0\ A1) \tag{10}$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{11}$$

Definition 15 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{12}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

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Definition 16 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 17 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 18 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 19 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(14)

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(15)

Definition 20 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(16)

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
(17)

Definition 21 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 22 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 23 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 24 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 25 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap\ ($c_2Efcp_2EFC

Definition 26 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 27 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ ($c_2Eboo

Let $c\_2Esum\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum})$$
(18)

Definition 28 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Definition 29 *We define* c_2Ewords_2Eword__add *to be* $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\\ \mathsf{c\_2Enum\_2E0}) = V0m)) \tag{19}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\\ V1n)\ V0m)))) \tag{20}$$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in\\ A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Rightarrow}True){\Leftrightarrow}\\ True)\wedge(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge(((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge((\\ (p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t))))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((V0x = V0x){\Leftrightarrow}\\ True)) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in\\ A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{25}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1){\Rightarrow}\\ ((p\ V1t2){\Rightarrow}(p\ V2t3))){\Leftrightarrow}(((p\ V0t1)\wedge(p\ V1t2)){\Rightarrow}(p\ V2t3)))))) \tag{26}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in\\ 2.(((((p\ V0x){\Leftrightarrow}(p\ V1x\_27))\wedge((p\ V1x\_27){\Rightarrow}((p\ V2y){\Leftrightarrow}(p\ V3y\_27)))){\Rightarrow}\\ (((p\ V0x){\Rightarrow}(p\ V2y)){\Leftrightarrow}((p\ V1x\_27){\Rightarrow}(p\ V3y\_27)))))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\\ \forall V0x \in (ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b).(\forall V1y \in (ty\_2Efcp\_2Ecart\\ A\_27a\ A\_27b).((V0x = V1y){\Leftrightarrow}(\forall V2i \in ty\_2Enum\_2Enum.((p\ (ap\\ (ap\ \mathsf{c\_2Eprim\_rec\_2E\_3C}\ V2i)\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27b)\ (\\ c\_2Ebool\_2Ethe\_\_value\ A\_27b)))){\Rightarrow}((ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\\ A\_27a\ A\_27b)\ V0x)\ V2i) = (ap\ (ap\ (\mathsf{c\_2Efcp\_2Efcp\_\_index}\ A\_27a\ A\_27b)\\ V1y)\ V2i)))))))) \tag{28}$$

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Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart$$
$$2 \ A\_27a).(\exists V1n \in ty\_2Enum\_2Enum.((V0w = (ap \ (\textsf{c\_2Ewords\_2En2w}$$
$$A\_27a) \ V1n)) \wedge (p \ (ap \ (ap \ \textsf{c\_2Eprim\_rec\_2E\_3C} \ V1n) \ (ap \ (c\_2Ewords\_2Edimword$$
$$A\_27a) \ (c\_2Ebool\_2Ethe\_value \ A\_27a)))))))$$

$$(29)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.($$
$$\forall V1n \in ty\_2Enum\_2Enum.((ap \ (ap \ (\textsf{c\_2Ewords\_2Eword\_add}$$
$$A\_27a) \ (ap \ (\textsf{c\_2Ewords\_2En2w} \ A\_27a) \ V0m)) \ (ap \ (\textsf{c\_2Ewords\_2En2w}$$
$$A\_27a) \ V1n)) = (ap \ (\textsf{c\_2Ewords\_2En2w} \ A\_27a) \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B$$
$$V0m) \ V1n)))))$$

$$(30)$$

Theorem 1

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow ((\forall V0w \in (ty\_2Efcp\_2Ecart$$
$$2 \ A\_27a).((ap \ (ap \ (\textsf{c\_2Ewords\_2Eword\_add} \ A\_27a) \ V0w) \ (ap \ (\textsf{c\_2Ewords\_2En2w}$$
$$A\_27a) \ \textsf{c\_2Enum\_2E0})) = V0w)) \wedge (\forall V1w \in (ty\_2Efcp\_2Ecart \ 2$$
$$A\_27a).((ap \ (ap \ (\textsf{c\_2Ewords\_2Eword\_add} \ A\_27a) \ (ap \ (\textsf{c\_2Ewords\_2En2w}$$
$$A\_27a) \ \textsf{c\_2Enum\_2E0})) \ V1w = V1w)))$$