

thm_2Ewords_2EWORD__ADD__LID__UNIQ
(TMLsufaeKHsBx8qWGYGDhfbokPzZue6yNq4)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Definition 8 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 10 We define $c_Ebit_EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_Earithmetic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 11 We define $c_Ebit_EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define c_Ebit_EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 13 We define c_Ebit_EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efc_2Efinite_image A0) \quad (11)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (12)$$

Let $c_Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Efc_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Definition 14 We define c_Ebool_2EF to be $(ap (c_Ebool_2E.21 2) (\lambda V0t \in 2.V0t))$.

Definition 15 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 16 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 17 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 18 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A.\lambda P x))$ of type $\iota \Rightarrow \iota$.

Definition 19 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a))))$

Definition 20 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (c_2Eprim_rec_2E_3C V0m) V1n))$

Definition 21 We define `c_2Ebool_2E_3F_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a) V0P))$

Definition 22 We define `c_2Efcp_2Efinite_index` to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 A_27a)^{ty_2Enum_2Enum})$

Let `ty_2Efcp_2Ecart` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (15)$$

Let `c_2Efcp_2Edest_cart` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (16)$$

Definition 23 We define `c_2Efcp_2Efcp_index` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b)$

Definition 24 We define `c_2Efcp_2EFCP` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (ap (c_2Efcp_2EFCP V0g) A_27b)))$

Definition 25 We define `c_2Ewords_2En2w` to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP A_27a) V0n)$

Definition 26 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_7E V2t2) V1t1))))$

Definition 27 We define `c_2Ebit_2ESBIT` to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E_7E V0b) V1n)))$

Let `c_2Esum_num_2ESUM` : ι be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum)^{(ty_2Enum_2Enum)^{ty_2Enum_2Enum}})^{ty_2Enum_2Enum} \quad (17)$$

Definition 28 We define `c_2Ewords_2Ew2n` to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart 2 A_27a).(ap (ap (c_2Ewords_2Ew2n V0w) A_27a))$

Let `c_2Ewords_2Edimword` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2Edimword A_27a \in (ty_2Enum_2Enum)^{(ty_2Ebool_2Eitsself A_27a)} \quad (18)$$

Definition 29 We define $c_2Ewords_2Eword_comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 30 We define $c_2Ewords_2Eword_add$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a). \lambda V1w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 31 We define $c_2Ewords_2Eword_sub$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a). \lambda V1w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Efc_2Ecart\ 2\ A_27a). ((ap\ (ap\ (c_2Ewords_2Eword_sub\ A_27a)\ V0w)\ V0w) = (ap\ (c_2Ewords_2En2w\ A_27a)\ c_2Enum_2E0)))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2Efc_2Ecart\ 2\ A_27a). (\forall V1w \in (ty_2Efc_2Ecart\ 2\ A_27a). (\forall V2x \in (ty_2Efc_2Ecart\ 2\ A_27a). ((ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ V0v)\ V1w) = V2x) \Leftrightarrow (V0v = (ap\ (ap\ (c_2Ewords_2Eword_sub\ A_27a)\ V2x)\ V1w))))))) \quad (20)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2Efc_2Ecart\ 2\ A_27a). (\forall V1w \in (ty_2Efc_2Ecart\ 2\ A_27a). (((ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ V0v)\ V1w) = V1w) \Leftrightarrow (V0v = (ap\ (c_2Ewords_2En2w\ A_27a)\ c_2Enum_2E0)))))))$$