

thm_2Ewords_2EWORD__ADD__RIGHT__LO
(TMKENAh-
nyBMwsyRUkx1TeQuuJxRtFoVnxB)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2EBOUNDED$ to be $(\lambda V0v \in 2.c_2Ebool_2E_2T)$.

Let $ty_2Efcf_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efcf_2Efinite_image A0) \quad (1)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (2)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (3)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (4)$$

Let $c_2Efcf_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcf_2Edimindex A_27a \in (ty_2Eenum_2Eenum^{(ty_2Ebool_2Eitself A_27a)}) \quad (5)$$

Definition 7 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E))$

Definition 8 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num)$

Definition 10 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_2E_40$

Definition 12 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_Ebool_2E_2F_5C$

Definition 14 We define $c_Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40 (A_27a^{ty_2Enum_2Enum}))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (9)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (10)$$

Definition 15 We define $c_Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 16 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (12)$$

Definition 18 We define $c_Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E_2B))$

Definition 19 We define $c_Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 20 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_Ebool_2ECOND A_27a t1 t2))))$

Definition 21 We define c_Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap c_Ebool_2ECOND A_27a)))$

Let $c_Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 22 We define c_Ewords_2Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap (ap c_Esum_num_2ESUM A_27a w))$

Let $c_Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)})^{ty_2Enum_2Enum} \quad (15)$$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Definition 23 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2E_2D))$

Let $c_Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (17)$$

Definition 24 We define $c_Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2EDIV A_27a))$

Let $c_Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (18)$$

Definition 25 We define $c_Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_2EMOD A_27a))$

Definition 26 We define c_Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V2n \in ty_2Enum_2Enum.(ap (ap (ap c_Ebit_2EBIT1 A_27a)))$

Definition 27 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 28 We define c_2EfcP_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 29 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2EfcP_2EFCP$

Definition 30 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).$

Definition 31 We define $c_2Ewords_2Eword_2msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap$

Definition 32 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21\ 2) (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (19)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (20)$$

Definition 33 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 34 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b$

Definition 35 We define $c_2Ewords_2Eenzcv$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1b \in ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (21)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (22)$$

Definition 36 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Definition 37 We define $c_2Ewords_2Eword_2lo$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1b$

Definition 38 We define $c_2Ewords_2Eword_2add$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1$

Definition 39 We define $c_2Ewords_2Eword_2ls$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1b$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge \neg(p V1B)))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. (\forall V3y \in A_27a. ((ap V0f (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1b) V2x) V3y)) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V1b) (ap V0f V2x)) (ap V0f V3y))))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0P \in (2^{A_27b}). (\forall V1N \in (A_27b^{A_27a}). (\forall V2M \in A_27a. ((p (ap V0P (ap (ap (c_2Ebool_2ELET A_27a A_27b) (\lambda V3x \in A_27a. (ap V1N V3x))) V2M))) \Leftrightarrow (p (ap (ap (c_2Ebool_2ELET A_27a 2) (\lambda V4x \in A_27a. (ap V0P (ap V1N V4x))) V2M)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) V5y_27)))))) \quad (30)$$

Assume the following.

$$(\forall V0v \in 2.((p (ap c_2Ebool_2EBOUNDED V0v)) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0a \in (ty_2Efc_2Ecart \\ 2 A_27a).(\forall V1b \in (ty_2Efc_2Ecart 2 A_27a).((\neg(p (ap (\\ ap (c_2Ewords_2Eword_ls A_27a) V0a) V1b)))) \Leftrightarrow (p (ap (ap (c_2Ewords_2Eword_ls \\ A_27a) V1b) V0a)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Efc_2Ecart \\ 2 A_27a).(\forall V1c \in (ty_2Efc_2Ecart 2 A_27a).(\forall V2a \in \\ (ty_2Efc_2Ecart 2 A_27a).((p (ap (ap (c_2Ewords_2Eword_ls \\ A_27a) (ap (ap (c_2Ewords_2Eword_add A_27a) V2a) V0b)) V1c)) \Leftrightarrow \\ (p (ap (ap (ap (c_2Ebool_2ECOND 2) (ap (ap (c_2Ewords_2Eword_ls \\ A_27a) V0b) V1c)) (ap (ap (c_2Ebool_2ELET (ty_2Efc_2Ecart 2 A_27a) \\ 2) (\lambda V3x \in (ty_2Efc_2Ecart 2 A_27a).(ap (ap c_2Ebool_2E_5C_2F \\ (ap (ap (c_2Ewords_2Eword_ls A_27a) V2a) V3x)) (ap (ap c_2Ebool_2E_2F_5C \\ (ap c_2Ebool_2E_7E (ap (ap (c_2Emin_2E_3D (ty_2Efc_2Ecart 2 \\ A_27a) V0b) (ap (c_2Ewords_2Een2w A_27a) c_2Enum_2E0)))) (ap (\\ ap (c_2Ewords_2Eword_ls A_27a) (ap (ap (c_2Ewords_2Eword_add \\ A_27a) (ap (c_2Ewords_2Eword_2comp A_27a) V1c)) V3x)) V2a)))) \\ (ap (c_2Ewords_2Een2w A_27a) (ap (ap c_2Earithmetic_2E_2D (ap (\\ c_2Ewords_2Ew2n A_27a) V1c)) (ap (c_2Ewords_2Ew2n A_27a) V0b)))) \\ (ap (ap c_2Ebool_2E_2F_5C (ap (ap (c_2Ewords_2Eword_ls A_27a) \\ (ap (c_2Ewords_2Eword_2comp A_27a) V0b)) V2a)) (ap (ap (c_2Ewords_2Eword_ls \\ A_27a) V2a) (ap (ap (c_2Ewords_2Eword_add A_27a) (ap (c_2Ewords_2Eword_2comp \\ A_27a) V0b)) V1c)))))))))) \end{aligned} \quad (33)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0c \in (ty_2EfcP_2Ecart \\ & \quad 2\ A_27a).(\forall V1a \in (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V2b \in \\ & \quad (ty_2EfcP_2Ecart\ 2\ A_27a).((p\ (ap\ (ap\ (c_2Ewords_2Eword_lo \\ & \quad A_27a)\ V1a)\ (ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ V2b)\ V0c)))) \Leftrightarrow \\ & \quad (p\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ 2)\ (ap\ (ap\ (c_2Ewords_2Eword_ls \\ & \quad A_27a)\ V0c)\ V1a))\ (ap\ (ap\ (c_2Ebool_2ELET\ (ty_2EfcP_2Ecart\ 2\ A_27a) \\ & \quad 2)\ (\lambda V3x \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ c_2Ebool_2E_2F_5C \\ & \quad (ap\ (ap\ (c_2Ewords_2Eword_lo\ A_27a)\ V3x)\ V2b))\ (ap\ (ap\ c_2Ebool_2E_5C_2F \\ & \quad (ap\ (ap\ (c_2Emin_2E_3D\ (ty_2EfcP_2Ecart\ 2\ A_27a))\ V0c)\ (ap\ (c_2Ewords_2Een2w \\ & \quad A_27a)\ c_2Enum_2E0))))\ (ap\ (ap\ (c_2Ewords_2Eword_lo\ A_27a)\ V2b) \\ & \quad (ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp \\ & \quad A_27a)\ V1a))\ V3x))))))\ (ap\ (c_2Ewords_2Een2w\ A_27a)\ (ap\ (ap\ c_2Earithmetic_2E_2D \\ & \quad (ap\ (c_2Ewords_2Ew2n\ A_27a)\ V1a))\ (ap\ (c_2Ewords_2Ew2n\ A_27a) \\ & \quad V0c))))\ (ap\ (ap\ c_2Ebool_2E_5C_2F\ (ap\ (ap\ (c_2Ewords_2Eword_lo \\ & \quad A_27a)\ V2b)\ (ap\ (c_2Ewords_2Eword_2comp\ A_27a)\ V0c)))\ (ap\ (ap \\ & \quad (c_2Ewords_2Eword_lo\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_add \\ & \quad A_27a)\ (ap\ (c_2Ewords_2Eword_2comp\ A_27a)\ V0c))\ V1a))\ V2b))))))))) \end{aligned}$$