

thm_2Ewords_2EWORD__ADD__SUB3
(TMTHW61RANqhejAmSK2Nr84QdrDdYneWAbD)

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Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (2)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (3)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (5)$$

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 4 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))$
Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 13 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (9)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (10)$$

Definition 14 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 16 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 17 We define `c.Earithmic.EBIT2` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c.Earithmic$

Definition 18 We define `c.Earithmic.ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `c.Earithmic.EEXP` : ι be given. Assume the following.

$$c.Earithmic.EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 19 We define `c.Ebool.ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 20 We define `c.Ebit.ESBIT` to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c.Ebo$

Let `c.Esum_num.ESUM` : ι be given. Assume the following.

$$c.Esum_num.ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 21 We define `c.Ewords.Ew2n` to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap (ap$

Definition 22 We define `c.Earithmic.EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c.Earithmic$

Let `c.Earithmic.EDIV` : ι be given. Assume the following.

$$c.Earithmic.EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Definition 23 We define `c.Ebit.EDIV_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let `c.Earithmic.E_2D` : ι be given. Assume the following.

$$c.Earithmic.E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Let `c.Earithmic.EMOD` : ι be given. Assume the following.

$$c.Earithmic.EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Definition 24 We define `c.Ebit.EMOD_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define `c.Ebit.EBITS` to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 26 We define `c.Ebit.EBIT` to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 27 We define `c.EfcP.EFCP` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 28 We define `c.Ewords.En2w` to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c.EfcP.EFC$

Definition 29 We define `c.Ewords.Eword_add` to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (18)$$

Definition 30 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 31 We define $c_2Ewords_2Eword_2sub$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_2sub\ A_27a)\ V0w)\ V0w) = (ap\ (c_2Ewords_2En2w\ A_27a)\ c_2Enum_2E0))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).(\forall V1w \in (ty_2Efc_2Ecart\ 2\ A_27a).(\forall V2x \in (ty_2Efc_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_2sub\ A_27a)\ V0v)\ (ap\ (ap\ (c_2Ewords_2Eword_2add\ A_27a)\ V1w)\ V2x)) = (ap\ (ap\ (c_2Ewords_2Eword_2sub\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_2sub\ A_27a)\ V0v)\ V1w))\ V2x)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_2sub\ A_27a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ c_2Enum_2E0))\ V0w) = (ap\ (c_2Ewords_2Eword_2comp\ A_27a)\ V0w))) \quad (21)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).(\forall V1x \in (ty_2Efc_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_2sub\ A_27a)\ V0v)\ (ap\ (ap\ (c_2Ewords_2Eword_2add\ A_27a)\ V0v)\ V1x)) = (ap\ (c_2Ewords_2Eword_2comp\ A_27a)\ V1x))))$$