

thm_2Ewords_2EWORD__BITS__COMP__THM
(TMFHosYy5bJZnXXVFWWqEENSg3gcZeGcfP6)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \rightarrow \iota$.

Definition 2 We define c_Ebool_ET to be $(ap \ (ap \ (c_Emin_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A._27a : \iota.(\lambda V0P \in (2^A_{27}a)).(ap\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^A_{27}a)\ V)\ P)\ 0)\ P)$

Definition 4 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in 2.$

Definition 7 We define $c_{\text{Emin}} \in E_40$ to be $\lambda A. \lambda P \in 2^A . \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota)$.

Definition 8 We define c_2Ebool_2ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.(a$

Definition 9 We define $c_2Emarker_2EAC$ to be $\lambda V0b1 \in 2.\lambda V1b2 \in 2.(ap\ (ap\ c_2Ebool_2E_2F_5C\ V0b1)\ V1b2)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

nonempty *ty_2Enum_2Enum* (1)

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Earithmic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EOOD \in (2^{ty_2Enum_2Enum}) \quad (3)$$

Definition 10 We define $c_{\text{Ebool_2E_7E}}$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_{\text{Emin_2E_3D_3D_3E}}\ V0t)\ c_{\text{Ebool_2E_7E}}))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 13 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 14 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 15 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 16 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 17 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP).$

Definition 18 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E_21 2) (\lambda V1n \in ty_2Enum_2Enum.\lambda V2n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 19 We define $c_2Enumeral_2EiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 20 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x.$

Definition 21 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2BIT2 (\lambda V1n \in ty_2Enum_2Enum.\lambda V2n \in ty_2Enum_2Enum.$

Definition 22 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (11)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2Edimword A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (12)$$

Definition 23 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 24 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 25 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ebool_2Ethethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (13)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcp_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (14)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Definition 26 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Definition 27 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Definition 28 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 29 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \lambda V$

Definition 30 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.(ap$

Let $ty_2Efcp_2Efinit_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2Efcp_2Efinit_image A0) \quad (18)$$

Definition 31 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 32 We define $c_2Efcp_2Efinit_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Efcp_2Ecart \\ A0 A1) \end{aligned} \quad (19)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Efcp_2Edest_cart \\ A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinit_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \end{aligned} \quad (20)$$

Definition 33 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b).$

Definition 34 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcp_2EFCP A_27a A_27b)))$

Definition 35 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP A_27a A_27b)))$

Definition 36 We define $c_2Ewords_2Eword_bits$ to be $\lambda A_27a : \iota. \lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \\ & (21) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \\ & (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \\ & (23) \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
 & (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \\
 \end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap c_2Enum_2ESUC V0m)) V1n)))))) \\
 \end{aligned} \tag{25}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 c_2Enum_2E0) V0n))) \tag{26}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V1n) V0m)))))) \\
 \end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\
 & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
 & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
 & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
 & V0m) V1n)))))))))) \\
 \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
 & ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \\
 \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
 & V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \\
 & (30)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
 & V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
 & V1n)) V0m)))))) \\
 & (31)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
 & c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & c_2Earithmetic_2EZERO)) V0n))) \\
 & (32)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (ap (ap c_2Earithmetic_2EMIN V0m) V1n) = (ap (ap c_2Earithmetic_2EMIN \\
 & V1n) V0m)))) \\
 & (33)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EMIN V0m) \\
 & (ap (ap c_2Earithmetic_2EMIN V1n) V2p)) = (ap (ap c_2Earithmetic_2EMIN \\
 & (ap (ap c_2Earithmetic_2EMIN V0m) V1n)) V2p)))))) \\
 & (34)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h1 \in ty_2Enum_2Enum. (\forall V1l1 \in ty_2Enum_2Enum. \\
 & (\forall V2h2 \in ty_2Enum_2Enum. (\forall V3l2 \in ty_2Enum_2Enum. \\
 & (\forall V4n \in ty_2Enum_2Enum. ((ap (ap (ap c_2Ebit_2EBITS V2h2) \\
 & V3l2) (ap (ap (ap c_2Ebit_2EBITS V0h1) V1l1) V4n)) = (ap (ap (ap c_2Ebit_2EBITS \\
 & (ap (ap c_2Earithmetic_2EMIN V0h1) (ap (ap c_2Earithmetic_2E_2B \\
 & V2h2) V1l1))) (ap (ap c_2Earithmetic_2E_2B V3l2) V1l1)) V4n))))))) \\
 & (35)
 \end{aligned}$$

Assume the following.

$$True \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
 V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0t \in 2. ((\forall V1x \in \\ A_27a. (p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow \\ (p V0t)) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (43)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \quad (44)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ True)) \quad (45)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (46)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \\ V0t))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ A_{.27a}.(((ap\ (ap\ (ap\ (c_{.2Ebool_2ECOND}\ A_{.27a})\ c_{.2Ebool_2ET})\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_{.2Ebool_2ECOND}\ A_{.27a})\ c_{.2Ebool_2EF}) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (\\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((((\neg(p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee (\neg(p\ V1B)))) \wedge ((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B)))))))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ (p\ V1B)))))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (52)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))))) \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ \forall V0b \in 2.(\forall V1f \in (A_{.27b}^{A_{.27a}}).(\forall V2g \in (A_{.27b}^{A_{.27a}}). \\ (\forall V3x \in A_{.27a}.((ap\ (ap\ (ap\ (c_{.2Ebool_2ECOND}\ (A_{.27b}^{A_{.27a}})) \\ V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c_{.2Ebool_2ECOND}\ A_{.27b})\ V0b)\ (ap \\ V1f\ V3x))\ (ap\ V2g\ V3x))))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ \forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1b \in 2.(\forall V2x \in A_{.27a}. \\ (\forall V3y \in A_{.27a}.((ap\ V0f)\ (ap\ (ap\ (ap\ (c_{.2Ebool_2ECOND}\ A_{.27a}) \\ V1b)\ V2x)\ V3y) = (ap\ (ap\ (ap\ (c_{.2Ebool_2ECOND}\ A_{.27b})\ V1b)\ (ap\ V0f \\ V2x))\ (ap\ V0f\ V3y))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned}
 & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
 & (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\
 & (\forall V5y_27 \in A_27a.(((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
 & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
 & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
 & V5y_27))))))) \\
 \end{aligned} \tag{56}$$

Assume the following.

$((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \wedge ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m))))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n))))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n))))))) \wedge ((\forall V19n \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum.((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V23n) c_2Enum_2E0)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.((\forall V26m \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V25n) c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V25n) V26m))))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E V28n) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n))))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.((\forall V30m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C V29n) c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V30m)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C V30m) V29n))))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow False))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{59}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{63}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\ & \end{aligned} \quad (67)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (68)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0w \in (\text{ty_2Efc}_2Ecart \\ & 2 A_27a). (\exists V1n \in \text{ty_2Enum_2Enum}. ((V0w = (\text{ap } (\text{c_2Ewords_2En2w} \\ & A_27a) V1n)) \wedge (p (\text{ap } (\text{ap } \text{c_2Eprim_rec_3C } V1n) (\text{ap } (\text{c_2Ewords_2Edimword} \\ & A_27a) (\text{c_2Ebool_2Eth}_value A_27a))))))) \\ & \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0h \in \text{ty_2Enum_2Enum}. (\\ & \forall V1l \in \text{ty_2Enum_2Enum}. (\forall V2n \in \text{ty_2Enum_2Enum}. (\\ & \text{ap } (\text{ap } (\text{ap } (\text{c_2Ewords_2Eword_bits } A_27a) V0h) V1l) (\text{ap } (\text{c_2Ewords_2En2w} \\ & A_27a) V2n)) = (\text{ap } (\text{c_2Ewords_2En2w } A_27a) (\text{ap } (\text{ap } \text{c_2Ebit_2EBITS} \\ & (\text{ap } (\text{ap } \text{c_2Earithmetic_2EMIN } V0h) (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2D} \\ & (\text{ap } (\text{c_2Efc}_2Edimindex A_27a) (\text{c_2Ebool_2Eth}_value A_27a))) \\ & (\text{ap } \text{c_2Earithmetic_2ENUMERAL } (\text{ap } \text{c_2Earithmetic_2EBIT1 } \text{c_2Earithmetic_2EZERO}))) \\ & V1l) V2n))))))) \\ & \end{aligned} \quad (70)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0h1 \in \text{ty_2Enum_2Enum}. \\ & (\forall V1l1 \in \text{ty_2Enum_2Enum}. (\forall V2h2 \in \text{ty_2Enum_2Enum}. \\ & (\forall V3l2 \in \text{ty_2Enum_2Enum}. (\forall V4w \in (\text{ty_2Efc}_2Ecart \\ & 2 A_27a). ((\text{ap } (\text{ap } (\text{ap } (\text{c_2Ewords_2Eword_bits } A_27a) V2h2) V3l2) \\ & (\text{ap } (\text{ap } (\text{ap } (\text{c_2Ewords_2Eword_bits } A_27a) V0h1) V1l1) V4w)) = (\\ & \text{ap } (\text{ap } (\text{ap } (\text{c_2Ewords_2Eword_bits } A_27a) (\text{ap } (\text{ap } \text{c_2Earithmetic_2EMIN} \\ & V0h1) (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2B } V2h2) V1l1))) (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2B} \\ & V3l2) V1l1)) V4w))))))) \\ & \end{aligned}$$