

thm_2Ewords_2EWORD__DIVISION (TM- SWTqt4TYmqoeGLjTLHRMYmUcA18GAb5h)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}}))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let `c_2Earithmetic_2EDIV` : ι be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let `ty_2Efc_2Efinite_image` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \tag{3}$$

Let `ty_2Ebool_2Eitself` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \tag{4}$$

Let `c_2Ebool_2Ethe_value` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Ebool_2Ethe_value\ A_{27a} \in (ty_2Ebool_2Eitself\ A_{27a}) \tag{5}$$

Let `c_2Efc_2Edimindex` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow c_2Efc_2Edimindex\ A_{27a} \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_{27a})}) \tag{6}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2))$
Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x))$
of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 13 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \quad (10)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \quad (11)$$

Definition 14 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27a\ A_27b)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Definition 15 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 16 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 17 We define $c_Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic$

Definition 18 We define $c_Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 19 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 20 We define c_Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_Ebo$

Let $c_Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (15)$$

Definition 21 We define c_Ewords_2Ew2n to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap (ap c$

Definition 22 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic$

Definition 23 We define $c_Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Let $c_Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (17)$$

Definition 24 We define $c_Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define c_Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 26 We define c_Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 27 We define c_EfcP_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 28 We define c_Ewords_2En2w to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_EfcP_2EFC$

Definition 29 We define $c_Ewords_2Eword_div$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1$

Definition 30 We define $c_Ewords_2Eword_mod$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V$

Let $c_Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)})^{ty_2Enum_2Enum} \quad (18)$$

Definition 31 We define $c_Ewords_Eword_add$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_Efc_Ecart\ 2\ A_27a).\lambda V$.
Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (19)$$

Definition 32 We define $c_Ewords_Eword_mul$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_Efc_Ecart\ 2\ A_27a).\lambda V$.

Definition 33 We define $c_Ewords_Eword_comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).$

Definition 34 We define $c_Ewords_Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).$

Definition 35 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in$

Let $ty_Epair_Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_Epair_Eprod\ A0\ A1) \quad (20)$$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EABS_prod\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (21)$$

Definition 36 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Definition 37 We define $c_Ebool_E_ELET$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27$

Definition 38 We define $c_Ewords_E_enzcv$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_Efc_Ecart\ 2\ A_27a).\lambda V1b \in ($

Let $c_Epair_EESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EESND\ A_27a\ A_27b \in (A_27b^{(ty_Epair_Eprod\ A_27a\ A_27b)}) \quad (22)$$

Let $c_Epair_E_EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_E_EFST\ A_27a\ A_27b \in (A_27a^{(ty_Epair_Eprod\ A_27a\ A_27b)}) \quad (23)$$

Definition 39 We define $c_Epair_E_EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Definition 40 We define $c_Ewords_E_lo$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_Efc_Ecart\ 2\ A_27a).\lambda V1b$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((\neg(V0n = c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V1n) V2p))) \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V2p)))))) \quad (25)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow (\forall V1k \in ty_2Enum_2Enum.((V1k = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2EDIV V1k) V0n)) V0n)) (ap (ap c_2Earithmetic_2EMOD V1k) V0n))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C (ap (ap c_2Earithmetic_2EMOD V1k) V0n)) V0n)))))) \quad (26)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1k \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C V1k) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD V1k) V0n) = V1k))) \quad (27)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD c_2Enum_2E0) V0n) = c_2Enum_2E0))) \quad (28)$$

Assume the following.

$$True \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27}))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (p (ap (ap c_{2Eprim_rec_2E_3C} c_{2Enum_2E0}) \\
& (ap (c_{2Ewords_2Edimword} A_{27a}) (c_{2Ebool_2Ethe_value} A_{27a}))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0n \in ty_{2Enum_2Enum}.(\\
& (ap (c_{2Ewords_2En2w} A_{27a}) (ap (c_{2Ewords_2En2w} A_{27a}) V0n)) = \\
& (ap (ap c_{2Earithmetic_2EMOD} V0n) (ap (c_{2Ewords_2Edimword} A_{27a}) \\
& (c_{2Ebool_2Ethe_value} A_{27a}))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0m \in ty_{2Enum_2Enum}.(\\
& \forall V1n \in ty_{2Enum_2Enum}.(((ap (c_{2Ewords_2En2w} A_{27a}) V0m) = \\
& (ap (c_{2Ewords_2En2w} A_{27a}) V1n)) \Leftrightarrow ((ap (ap c_{2Earithmetic_2EMOD} \\
& V0m) (ap (c_{2Ewords_2Edimword} A_{27a}) (c_{2Ebool_2Ethe_value} \\
& A_{27a}))) = (ap (ap c_{2Earithmetic_2EMOD} V1n) (ap (c_{2Ewords_2Edimword} \\
& A_{27a}) (c_{2Ebool_2Ethe_value} A_{27a}))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0w \in (ty_{2EfcP_2Ecart} \\
& 2 A_{27a}).(\exists V1n \in ty_{2Enum_2Enum}.((V0w = (ap (c_{2Ewords_2En2w} \\
& A_{27a}) V1n)) \wedge (p (ap (ap c_{2Eprim_rec_2E_3C} V1n) (ap (c_{2Ewords_2Edimword} \\
& A_{27a}) (c_{2Ebool_2Ethe_value} A_{27a}))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\\ \forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ewords_2Eword_add \\ A_27a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ V0m))\ (ap\ (c_2Ewords_2En2w \\ A_27a)\ V1n)) = (ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ V0m)\ V1n)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\\ \forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ewords_2Eword_mul \\ A_27a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ V0m))\ (ap\ (c_2Ewords_2En2w \\ A_27a)\ V1n)) = (ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (ap\ c_2Earithmetic_2E_2A \\ V0m)\ V1n)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Efc_2Ecart \\ 2\ A_27a).(\forall V1b \in (ty_2Efc_2Ecart\ 2\ A_27a).((p\ (ap\ (ap \\ (c_2Ewords_2Eword_lo\ A_27a)\ V0a)\ V1b)) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ (ap\ (c_2Ewords_2Ew2n\ A_27a)\ V0a))\ (ap\ (c_2Ewords_2Ew2n\ A_27a) \\ V1b)))))) \end{aligned} \quad (42)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Efc_2Ecart \\ 2\ A_27a).((\neg(V0w = (ap\ (c_2Ewords_2En2w\ A_27a)\ c_2Enum_2E0))) \Rightarrow \\ (\forall V1v \in (ty_2Efc_2Ecart\ 2\ A_27a).((V1v = (ap\ (ap\ (c_2Ewords_2Eword_add \\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_div \\ A_27a)\ V1v)\ V0w))\ V0w))\ (ap\ (ap\ (c_2Ewords_2Eword_mod\ A_27a)\ V1v) \\ V0w))) \wedge (p\ (ap\ (ap\ (c_2Ewords_2Eword_lo\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mod \\ A_27a)\ V1v)\ V0w))\ V0w)))))) \end{aligned}$$