

thm_2Ewords_2EWORD__EXTRACT__OVER__MUL2 (TMWTFY7TxduJvsX9EBVihFfEKCsDSfgY9E6)

October 26, 2020

Let $ty_2Efcp_2Ef\infty_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2Efcp_2Ef\infty_image A0) \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2Ebool_2Eitself A0) \quad (2)$$

Let $c_2Ebool_2Eth\infty_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ebool_2Eth\infty_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (3)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (4)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Efcp_2Edimindex A_27a \in (ty_2Enum_2Enum)^{(ty_2Ebool_2Eitself A_27a)} \quad (5)$$

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.(\lambda V3t3 \in 2.(ap (c_2Ebool_2E_7E V3t3) c_2Ebool_2EF)))))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num m)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) P)))$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (c_2Eprim_rec_2E_3C m n) m)$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a) P)))$

Definition 13 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum})))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Efcp_2Ecart A0 A1) \quad (9)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Efcp_2Edest_cart \\ & A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efcp_index A_27b)}))^{(ty_2Efcp_2Ecart A_27a A_27b)} \end{aligned} \quad (10)$$

Definition 14 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b).(\text{nonempty } (ty_2Efcp_2Edest_cart A_27a A_27b)))$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (12)$$

Definition 15 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (c_2Earithmetic_2E_3E m n) m)$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in$

Definition 17 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Definition 18 We define c_2Enum_2E0 to be $(ap(c_2Enum_2EABS_num c_2Enum_2EZERO_REP)).$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 20 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap(ap(ap(c_2Ebool_2E$

Definition 21 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap(c_2Enum_2ESUC)(ap$

Definition 22 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x.$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 23 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap(ap(c_2Earithmetic$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Definition 24 We define $c_2Earithmetic_2EZERO$ to be $c_2Enum_2E0.$

Definition 25 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap(ap(c_2Earithmetic$

Definition 26 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x.$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Definition 27 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 28 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 29 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \lambda V2l \in ty_2Enum_2Enum.$

Definition 30 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 31 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ewords_2Edimword A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)})$$

(19)

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$

(20)

Definition 32 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2. \lambda V1n \in ty_2Enum_2Enum.(ap$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})$$

(21)

Definition 33 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efcp_2Ecart 2 A_27a).(ap$

Definition 34 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum.(ap$

Definition 35 We define $c_2Ewords_2Eword_mul$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efcp_2Ecart 2 A_27a). \lambda V1n \in ty_2Enum_2Enum.$

Definition 36 We define $c_2Earithmetic_2EMIN$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 37 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 38 We define $c_2Ewords_2Eword_bits$ to be $\lambda A_27a : \iota. \lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum.$

Definition 39 We define $c_2Ewords_2Ew2w$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w \in (ty_2Efcp_2Ecart 2 A_27a). \lambda V1n \in ty_2Enum_2Enum.$

Definition 40 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1n \in ty_2Enum_2Enum.$

Definition 41 We define $c_2Ewords_2Eword_extract$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0h \in ty_2Enum_2Enum.$

Assume the following.

$$((ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) =$$

$$(ap c_2Enum_2ESUC c_2Enum_2E0))$$

(22)

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m))$$

(23)

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\
 & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\
 & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\
 & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\
 & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))
 \end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
 & V1n) V0m)))
 \end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
 & V1n) V0m)))
 \end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
 & (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) V2p))))
 \end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap c_2Enum_2ESUC V0m)) V1n))))
 \end{aligned} \tag{28}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 c_2Enum_2E0) V0n))) \tag{29}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V1n) V0m))))
 \end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n)))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{32}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \tag{33} \\
V0m) V0m)))$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Enum_2ESUC V1n)) V0m))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
 & V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
 & V1n)) V0m))))))) \\
 \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
 & c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & c_2Earithmetic_2EZERO))) V0n))) \\
 \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & (ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))))))) \\
 \end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V0m) \\
 & (ap (ap c_2Earithmetic_2E_2D V1n) V2p))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V2p)) V1n))))))) \\
 \end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C (\\
 & ap (ap c_2Earithmetic_2E_2D V0m) V1n)) V2p)) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p))) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
 & c_2Enum_2E0) V2p))))))) \\
 \end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. \\
 & (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\
 & V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B \\
 & V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\
 & V2b) V3d)) \Rightarrow (p (ap V0P V3d))))))) \\
 \end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
 & \quad (\forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad (ap (ap c_2Earithmetic_2EMIN V1m) V0n)) V2p)) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad (V1m) V2p)) \vee (p (ap (ap c_2Eprim_rec_2E_3C V0n) V2p)))) \wedge ((p (ap \\
 & \quad (ap c_2Eprim_rec_2E_3C V2p) (ap (ap c_2Earithmetic_2EMIN V1m) \\
 & \quad V0n))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C V2p) V1m)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
 & \quad V2p) V0n)))))))
 \end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
 & \quad (\forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & \quad (ap (ap c_2Earithmetic_2EMIN V1m) V0n)) V2p)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & \quad V1m) V2p)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D V0n) V2p)))) \wedge ((p \\
 & \quad (ap (ap c_2Earithmetic_2E_3C_3D V2p) (ap (ap c_2Earithmetic_2EMIN \\
 & \quad V1m) V0n))) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V2p) V1m)) \wedge (p \\
 & \quad (ap (ap c_2Earithmetic_2E_3C_3D V2p) V0n)))))))
 \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
 & \quad (ap (ap (ap c_2Ebit_2EBITS V0h) c_2Enum_2E0) V1n) = (ap (ap c_2Earithmetic_2EMOD \\
 & \quad V1n) (ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL \\
 & \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap c_2Enum_2ESUC \\
 & \quad V0h))))))
 \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty_2Enum_2Enum. (\forall V1a \in ty_2Enum_2Enum. \\
 & \quad (p (ap (ap c_2Eprim_rec_2E_3C V1a) (ap (ap c_2Earithmetic_2EEEXP \\
 & \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
 & \quad (ap c_2Enum_2ESUC V0h)))) \Rightarrow ((ap (ap (ap c_2Ebit_2EBITS V0h) c_2Enum_2E0) \\
 & \quad V1a) = V1a)))
 \end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h1 \in ty_2Enum_2Enum. (\forall V1l1 \in ty_2Enum_2Enum. \\
 & \quad (\forall V2h2 \in ty_2Enum_2Enum. (\forall V3l2 \in ty_2Enum_2Enum. \\
 & \quad (\forall V4n \in ty_2Enum_2Enum. ((ap (ap (ap c_2Ebit_2EBITS V2h2) \\
 & \quad V3l2) (ap (ap (ap c_2Ebit_2EBITS V0h1) V1l1) V4n)) = (ap (ap (ap c_2Ebit_2EBITS \\
 & \quad (ap (ap c_2Earithmetic_2EMIN V0h1) (ap (ap c_2Earithmetic_2E_2B \\
 & \quad V2h2) V1l1))) (ap (ap c_2Earithmetic_2E_2B V3l2) V1l1)) V4n)))))))
 \end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty_2Enum_2Enum. (\forall V1a \in ty_2Enum_2Enum. (\\
 & \forall V2b \in ty_2Enum_2Enum. ((ap (ap (ap c_2Ebit_2EBITS V0h) c_2Enum_2E0) \\
 & (ap (ap c_2Earithmetic_2E_2A (ap (ap (ap c_2Ebit_2EBITS V0h) c_2Enum_2E0) \\
 & V1a)) (ap (ap (ap c_2Ebit_2EBITS V0h) c_2Enum_2E0) V2b))) = (ap (\\
 & ap (ap c_2Ebit_2EBITS V0h) c_2Enum_2E0) (ap (ap c_2Earithmetic_2E_2A \\
 & V1a) V2b)))))) \\
 \end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty_2Enum_2Enum. (\forall V1l \in ty_2Enum_2Enum. (\\
 & \forall V2a \in ty_2Enum_2Enum. (\forall V3b \in ty_2Enum_2Enum. (\\
 & \forall V4x \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V1l) V4x)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V4x) V0h))) \Rightarrow ((p \\
 & (ap (ap c_2Ebit_2EBIT V4x) V2a)) \Leftrightarrow (p (ap (ap c_2Ebit_2EBIT V4x) V3b)))) \Leftrightarrow \\
 & ((ap (ap (ap c_2Ebit_2EBITS V0h) V1l) V2a) = (ap (ap (ap c_2Ebit_2EBITS \\
 & V0h) V1l) V3b)))))) \\
 \end{aligned} \tag{49}$$

Assume the following.

$$True \tag{50}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{52}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{53}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V0t))) \tag{54}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\
 & ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \\
 \end{aligned} \tag{55}$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (60)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \quad (61)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (62)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & (p V0t)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0b \in 2. (\forall V1t \in A_{\text{27a}}. \\ & ((ap (ap (ap (c_{\text{2Ebool_2ECOND}} A_{\text{27a}}) V0b) V1t) V1t) = V1t))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in (2^{A_{\text{27a}}}). ((\neg(\forall V1x \in \\ A_{\text{27a}}. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_{\text{27a}}. (\neg(p (ap V0P V2x))))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in (2^{A_{\text{27a}}}). (\forall V1Q \in \\ 2. (((\forall V2x \in A_{\text{27a}}. (p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in \\ A_{\text{27a}}. ((p (ap V0P V3x)) \wedge (p V1Q))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in (2^{A_{\text{27a}}}). (\forall V1Q \in \\ 2. (((\exists V2x \in A_{\text{27a}}. (p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in \\ A_{\text{27a}}. ((p (ap V0P V3x)) \vee (p V1Q))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_{\text{27a}}}). ((\exists V2x \in A_{\text{27a}}. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A_{\text{27a}}. (p (ap V1Q V3x))))) \end{aligned} \quad (70)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0Q \in 2. (\forall V1P \in (\\ 2^{A_{\text{27a}}}). ((\forall V2x \in A_{\text{27a}}. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in \\ A_{\text{27a}}. ((p (ap V1P V3x)) \vee (p V0Q))))) \end{aligned} \quad (71)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (\\ (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (\\ (p V0A)))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(\\ p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (74)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (75)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (76)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (77)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (78)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27))))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27))))))))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\ & (\forall V0P \in ((2^{A_27b})^{A_27a}). ((\forall V1x \in A_27a. (\exists V2y \in A_27b. \\ & (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_27b)^{A_27a}. \\ & (\forall V4x \in A_27a. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\
 & \forall V0x \in (ty_2Efcp_2Ecart\ A_{27a}\ A_{27b}).(\forall V1y \in (ty_2Efcp_2Ecart\ A_{27a}\ A_{27b}).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_2Enum_2Enum.((p (ap \\
 & (ap\ c_2Eprim_rec_2E_3C\ V2i)\ (ap\ (c_2Efcp_2Edimindex\ A_{27b})\ (\\
 & c_2Ebool_2Ethe_value\ A_{27b})))))) \Rightarrow ((ap\ (ap\ (c_2Efcp_2Efcp_index \\
 & A_{27a}\ A_{27b})\ V0x)\ V2i) = (ap\ (ap\ (c_2Efcp_2Efcp_index\ A_{27a}\ A_{27b}) \\
 & V1y)\ V2i)))))) \\
 \end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\
 & \forall V0g \in (A_{27a}^{ty_2Enum_2Enum}).(\forall V1i \in ty_2Enum_2Enum. \\
 & ((p (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1i)\ (ap\ (c_2Efcp_2Edimindex\ A_{27b})\ (\\
 & c_2Ebool_2Ethe_value\ A_{27b})))))) \Rightarrow ((ap\ (ap\ (c_2Efcp_2Efcp_index \\
 & A_{27a}\ A_{27b})\ (ap\ (c_2Efcp_2EFCP\ A_{27a}\ A_{27b})\ V0g))\ V1i) = (ap\ V0g \\
 & V1i)))))) \\
 \end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
 & (((ap\ c_2Enum_2ESUC\ c_2Earithmetic_2EZERO) = (ap\ c_2Earithmetic_2EBIT1 \\
 & c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum.((ap \\
 & c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2 \\
 & V0n)))) \wedge (\forall V1n \in ty_2Enum_2Enum.((ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT2 \\
 & V1n)) = (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enum_2ESUC\ V1n))))))) \\
 \end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap \\
& (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& ((\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
(ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC \\
c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum. \\
& (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Enum_2ESUC V17n)))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
(ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& ((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
c_2Enum_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{88}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C \\
V0n) V0n)))) \tag{89}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{90}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{91}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{93}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \end{aligned} \quad (96)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (97)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (98)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (99)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow ((ap (c_2Ewords_2Edimword A_27a) \\ & (c_2Ebool_2Ethethe_value A_27a)) = (ap (ap c_2Earithmetic_2EEEXP \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\ & (ap (c_2Efcp_2Edimindex A_27a) (c_2Ebool_2Ethethe_value A_27a)))) \end{aligned} \quad (100)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\ & (ap (c_2Efcp_2Edimindex A_27a) (c_2Ebool_2Ethethe_value A_27a)))) \end{aligned} \quad (101)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\exists V0m \in ty_2Enum_2Enum. (\\ & (ap (c_2Efcp_2Edimindex A_27a) (c_2Ebool_2Ethethe_value A_27a)) = \\ & (ap c_2Enum_2ESUC V0m))) \end{aligned} \quad (102)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2EMOD V0n) (ap (c_2Ewords_2Edimword A_27a) \\ & (c_2Ebool_2Ethethe_value A_27a))) = (ap (ap (ap c_2Ebit_2EBITS (\\ & ap (ap c_2Earithmetic_2E_2D (ap (c_2Efcp_2Edimindex A_27a) (c_2Ebool_2Ethethe_value \\ & A_27a))) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) c_2Enum_2E0) V0n))) \end{aligned} \quad (103)$$

Assume the following.

$$\begin{aligned} \forall A_{_27a}.nonempty\ A_{_27a} \Rightarrow & (\forall V0w \in (ty_2Efcp_2Ecart \\ & 2\ A_{_27a}).(\exists V1n \in ty_2Enum_2Enum.((V0w = (ap\ (c_2Ewords_2En2w \\ & A_{_27a})\ V1n)) \wedge (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1n)\ (ap\ (c_2Ewords_2Edimword \\ & A_{_27a})\ (c_2Ebool_2Ethe_value\ A_{_27a}))))))) \end{aligned} \quad (104)$$

Assume the following.

$$\begin{aligned} \forall A_{_27a}.nonempty\ A_{_27a} \Rightarrow & (\forall V0m \in ty_2Enum_2Enum.(\\ & \forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ewords_2Eword_mul \\ & A_{_27a})\ (ap\ (c_2Ewords_2En2w\ A_{_27a})\ V0m))\ (ap\ (c_2Ewords_2En2w \\ & A_{_27a})\ V1n)) = (ap\ (c_2Ewords_2En2w\ A_{_27a})\ (ap\ (ap\ c_2Earithmetic_2E_2A \\ & V0m)\ V1n)))))) \end{aligned} \quad (105)$$

Assume the following.

$$\begin{aligned} \forall A_{_27a}.nonempty\ A_{_27a} \Rightarrow & \forall A_{_27b}.nonempty\ A_{_27b} \Rightarrow \\ & (\forall V0h \in ty_2Enum_2Enum.(\forall V1l \in ty_2Enum_2Enum.(\forall V2n \in \\ & ty_2Enum_2Enum.((ap\ (ap\ (c_2Ewords_2Eword_extract\ A_{_27a} \\ & A_{_27b})\ V0h)\ V1l)\ (ap\ (c_2Ewords_2En2w\ A_{_27a})\ V2n)) = (ap\ (ap\ (ap\ (\\ & c_2Ebool_2ECOND\ (ty_2Efcp_2Ecart\ 2\ A_{_27b}))\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & (ap\ (c_2Efcp_2Edimindex\ A_{_27b})\ (c_2Ebool_2Ethe_value\ A_{_27b}))) \\ & (ap\ (c_2Efcp_2Edimindex\ A_{_27a})\ (c_2Ebool_2Ethe_value\ A_{_27a})))) \\ & (ap\ (c_2Ewords_2En2w\ A_{_27b})\ (ap\ (ap\ (ap\ c_2Ebit_2EBITS\ (ap\ (ap\ c_2Earithmetic_2EMIN \\ & V0h)\ (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ (c_2Efcp_2Edimindex\ A_{_27a}) \\ & (c_2Ebool_2Ethe_value\ A_{_27a})))\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V1l)\ V2n))) \\ & (ap\ (c_2Ewords_2En2w\ A_{_27b})\ (ap\ (ap\ (ap\ c_2Ebit_2EBITS\ (ap\ (ap\ c_2Earithmetic_2EMIN \\ & (ap\ (ap\ c_2Earithmetic_2EMIN\ V0h)\ (ap\ (ap\ c_2Earithmetic_2E_2D \\ & (ap\ (c_2Efcp_2Edimindex\ A_{_27a})\ (c_2Ebool_2Ethe_value\ A_{_27a}))) \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \\ & V1l)))\ V2n)))))) \end{aligned} \quad (106)$$

Theorem 1

$$\begin{aligned}
 & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow \\
 & \forall V0a \in (ty_2Efcpc_2Ecart\ 2\ A_{27a}).(\forall V1b \in (ty_2Efcpc_2Ecart \\
 & 2\ A_{27a}).(\forall V2h \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
 & V2h)\ (ap\ (c_2Efcpc_2Edimindex\ A_{27a})\ (c_2Ebool_2Ethethe_value\ A_{27a})))))) \Rightarrow \\
 & ((ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract\ A_{27b}\ A_{27b})\ V2h)\ c_2Enum_2E0) \\
 & (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_{27b})\ (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract \\
 & A_{27a}\ A_{27b})\ V2h)\ c_2Enum_2E0)\ V0a))\ (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract \\
 & A_{27a}\ A_{27b})\ V2h)\ c_2Enum_2E0)\ V1b)))) = (ap\ (ap\ (ap\ (c_2Ewords_2Eword_extract \\
 & A_{27a}\ A_{27b})\ V2h)\ c_2Enum_2E0)\ (ap\ (ap\ (c_2Ewords_2Eword_mul \\
 & A_{27a})\ V0a)\ V1b))))))
 \end{aligned}$$