

thm_2Ewords_2EWORD_LITERAL_MULT (TMRu6ebjrzcxb62sexr8b5mGG7s3Z4tFy9j)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \tag{4}$$

Definition 7 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B))$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 12 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B))$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 13 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2EDIV))$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 14 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2EMOD))$

Definition 15 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum. \lambda V1l \in ty_2Enum_2Enum. \lambda V2m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ c_2Ebit_2EDIV_2EXP)))$

Definition 16 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Ebit_2EBITS))$

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (12)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (13)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (14)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcp_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (15)$$

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 18 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 19 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a) P)))$

Definition 20 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (c_2Eprim_rec_2E_3C V0m) V1n)$

Definition 21 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C A_27a) P)))$

Definition 22 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 A_27a) (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}))$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (16)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (17)$$

Definition 23 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a A_27b).(ap (c_2Efcp_2Efcp_index V0x) A_27b)$

Definition 24 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (c_2Efcp_2EFCP V0g) A_27b))$

Definition 25 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcp_2EFCP A_27a) V0n)$

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_2Ebool_2ECOND A_27a V1t1 V2t2))))$

Definition 27 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2ECOND A_27a) V0b) V1n))$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 28 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ (ap\ c$

Definition 29 We define $c_2Ewords_2Eword_mul$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (19)$$

Definition 30 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\\ & \forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ (c_2Ewords_2Eword_mul \\ & A_27a)\ (ap\ (c_2Ewords_2En2w\ A_27a)\ V0m))\ (ap\ (c_2Ewords_2En2w \\ & A_27a)\ V1n)) = (ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (ap\ c_2Earithmetic_2E_2A \\ & V0m)\ V1n)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Efc_2Ecart \\ & 2\ A_27a).((ap\ (c_2Ewords_2Eword_2comp\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp \\ & A_27a)\ V0w)) = V0w)) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\ & 2 A_{.27a}). (\forall V1w \in (ty_2EfcP_2Ecart 2 A_{.27a}). ((ap (c_2Ewords_2Eword_2comp \\ & A_{.27a}) (ap (ap (c_2Ewords_2Eword_mul A_{.27a}) V0v) V1w)) = (ap (ap \\ & (c_2Ewords_2Eword_mul A_{.27a}) (ap (c_2Ewords_2Eword_2comp \\ & A_{.27a}) V0v)) V1w)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\ & 2 A_{.27a}). (\forall V1w \in (ty_2EfcP_2Ecart 2 A_{.27a}). ((ap (c_2Ewords_2Eword_2comp \\ & A_{.27a}) (ap (ap (c_2Ewords_2Eword_mul A_{.27a}) V0v) V1w)) = (ap (ap \\ & (c_2Ewords_2Eword_mul A_{.27a}) V0v) (ap (c_2Ewords_2Eword_2comp \\ & A_{.27a}) V1w)))))) \end{aligned} \quad (28)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow (\\ & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap (c_2Ewords_2Eword_mul A_{.27a}) (ap (c_2Ewords_2En2w A_{.27a}) \\ & V0m)) (ap (c_2Ewords_2Eword_2comp A_{.27a}) (ap (c_2Ewords_2En2w \\ & A_{.27a}) V1n))) = (ap (c_2Ewords_2Eword_2comp A_{.27a}) (ap (c_2Ewords_2En2w \\ & A_{.27a}) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))) \wedge (\forall V2m \in \\ & ty_2Enum_2Enum. (\forall V3n \in ty_2Enum_2Enum. ((ap (ap (c_2Ewords_2Eword_mul \\ & A_{.27b}) (ap (c_2Ewords_2Eword_2comp A_{.27b}) (ap (c_2Ewords_2En2w \\ & A_{.27b}) V2m))) (ap (c_2Ewords_2Eword_2comp A_{.27b}) (ap (c_2Ewords_2En2w \\ & A_{.27b}) V3n))) = (ap (c_2Ewords_2En2w A_{.27b}) (ap (ap c_2Earithmetic_2E_2A \\ & V2m) V3n)))))) \end{aligned}$$