thm_2Ewords_2EWORD__LT__EQ__LO (TMZaT7uHdiyX9iFtL5tgZTpSopNjHKmPKyN)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (6)$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

Definition 7 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Definition 9 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Definition 10 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Definition 11 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (11)$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

Definition 12 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 13 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 16 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 18 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 19 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 20 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 21 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 22 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 23 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2E}$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 24 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{14}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{15}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{16}$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{17}$$

Definition 25 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}$

Definition 26 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{18}$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{19}$$

Definition 27 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in(ty\_2Efcp\_2Ecart\ A\_27$

Definition 28 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b\in 2.\lambda V1n\in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM:\iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM\in((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum})\tag{20}$$

Definition 29 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a:\iota.\lambda V0w\in(ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $c\_2Ewords\_2Edimword:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ewords\_2Edimword\ A\_27a\in(ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)})\tag{21}$$

Definition 30 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n\in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 31 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b\in ty\_2Enum\_2Enum.\lambda V1n\in ty\_2Enum\_2Enum.(ap$

Definition 32 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0g\in(A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 33 *We define* c_2Ewords_2En2w *to be* $\lambda A_27a:\iota.\lambda V0n\in ty_2Enum_2Enum.(ap\ ($c_2Efcp_2EFC

Definition 34 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A\_27a:\iota.\lambda V0w\in(ty\_2Efcp\_2Ecart\ 2\ A\_27a).$

Definition 35 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A\_27a:\iota.\lambda V0w\in(ty\_2Efcp\_2Ecart\ 2\ A\_27a).(a$

Let $ty\_2Epair\_2Eprod:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1)\tag{22}$$

Let $c\_2Epair\_2EABS\_\_prod:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b\in((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})\tag{23}$$

Definition 36 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Definition 37 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.(\lambda V0f\in(A\_27b^{A\_27a}).(\lambda V1x\in A\_27$

Definition 38 *We define* c_2Ewords_2Enzcv *to be* $\lambda A\_27a:\iota.\lambda V0a\in(ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b\in($

Let $c\_2Epair\_2ESND:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b\in(A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})\tag{24}$$

Let $c\_2Epair\_2EFST:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b\in(A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})\tag{25}$$

Definition 39 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 40 *We define* c_2Ewords_2Eword__lo *to be* $\lambda A\_27a:\iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b$

Definition 41 *We define* c_2Ewords_2Eword__lt *to be* $\lambda A\_27a:\iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b$

Definition 42 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 43 *We define* c_2Ewords_2Eword__le *to be* $\lambda A\_27a:\iota.\lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).\lambda V1b$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{26}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))))) \tag{27}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m) \land (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n)) \land ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))))))))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \land (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{29}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{30}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m))))) \quad (31)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \quad (32)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1k)\ V0n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n) = V1k)))) \quad (33)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EMOD\ \mathsf{c\_2Enum\_2E0})\ V0n) = \mathsf{c\_2Enum\_2E0}))) \quad (34)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0n))\Rightarrow((ap\ (ap\ c\_2Earithmetic\_2EDIV\ \mathsf{c\_2Enum\_2E0})\ V0n) = \mathsf{c\_2Enum\_2E0}))) \quad (35)$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V0x)\ V1y)))\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0x))\lor(V1y = \mathsf{c\_2Enum\_2E0}))))) \quad (36)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n)))) \quad (37)$$

Assume the following.

$$True \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{41}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{42}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{44}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{47}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{49}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{51}$$

Assume the following.

$$
\begin{gathered}
((\forall V0n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
\mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap} \\
(\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in \\
\mathit{ty\_2Enum\_2Enum}.(\forall V3m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V3m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Enumeral\_2EiZ}\ (\mathit{ap} \\
(\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2B}\ V2n)\ V3m))))))\wedge((\forall V4n \in \mathit{ty\_2Enum\_2Enum}. \\
((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge \\
((\forall V5n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A} \\
V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in \mathit{ty\_2Enum\_2Enum}. \\
(\forall V7m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A}\ (\\
\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V7m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2A} \\
V6n)\ V7m)))))\wedge((\forall V8n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D} \\
\mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in \mathit{ty\_2Enum\_2Enum}. \\
((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D}\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in \\
\mathit{ty\_2Enum\_2Enum}.(\forall V11m \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V11m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2E\_2D} \\
V10n)\ V11m)))))\wedge((\forall V12n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP} \\
\mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap} \\
(\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in \\
\mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ V14n)\ \mathsf{c\_2Enum\_2E0}) = \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge \\
((\forall V15n \in \mathit{ty\_2Enum\_2Enum}.(\forall V16m \in \mathit{ty\_2Enum\_2Enum}. \\
((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n)) \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Earithmetic\_2EEXP}\ V15n)\ V16m)))))\wedge(((\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC} \\
\mathsf{c\_2Enum\_2E0}) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in \mathit{ty\_2Enum\_2Enum}.(\\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \\
\mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in \mathit{ty\_2Enum\_2Enum}.((\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in \mathit{ty\_2Enum\_2Enum}. \\
(((\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge \\
((\forall V20n \in \mathit{ty\_2Enum\_2Enum}.((\mathsf{c\_2Enum\_2E0} = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in \mathit{ty\_2Enum\_2Enum}. \\
(\forall V22m \in \mathit{ty\_2Enum\_2Enum}.(((\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V21n) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge \\
((\forall V23n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow \mathit{False}))\wedge((\forall V24n \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V24n)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V24n))))\wedge((\forall V25n \in \mathit{ty\_2Enum\_2Enum}.(\forall V26m \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V25n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V25n)\ V26m)))))\wedge((\forall V27n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E} \\
\mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow \mathit{False}))\wedge((\forall V28n \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\
V28n))))\wedge((\forall V29n \in \mathit{ty\_2Enum\_2Enum}.(\forall V30m \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
V29n))\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\
V30m)\ V29n)))))\wedge((\forall V31n \in \mathit{ty\_2Enum\_2Enum}.((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\
\mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow \mathit{True}))\wedge((\forall V32n \in \mathit{ty\_2Enum\_2Enum}. \\
((p\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}
\end{gathered}
$$

Assume the following.

$$\begin{gathered}(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ ((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\Leftrightarrow False)\wedge \\ ((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO})\Leftrightarrow \\ False)\wedge(((\mathsf{c\_2Earithmetic\_2EZERO} = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\ V0n))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = \mathsf{c\_2Earithmetic\_2EZERO})\Leftrightarrow \\ False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\ V1m))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\ V1m))\Leftrightarrow False)\wedge((((ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\ V1m))\Leftrightarrow(V0n = V1m))\wedge(((ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n) = (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\ V1m))\Leftrightarrow(V0n = V1m)))))))))))\end{gathered} \tag{53}$$

Assume the following.

$$\begin{gathered}(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO}) \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\ V0n)\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow \\ (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow \\ (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow \\ (\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C} \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow \\ (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))))))))\end{gathered} \tag{54}$$

Assume the following.

$$\begin{gathered}(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ ((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V0n))\Leftrightarrow \\ True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n))\ \mathsf{c\_2Earithmetic\_2EZERO}))\Leftrightarrow False)\wedge \\ (((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\ V0n))\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V0n)\ V1m)))))))))))\end{gathered} \tag{55}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((ap\ (c\_2Ewords\_2Edimword\ A\_27a) (c\_2Ebool\_2Ethe\_\_value\ A\_27a)) = (ap\ (ap\ c\_2Earithmetic\_2EEXP (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))) (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0n \in ty\_2Enum\_2Enum.((ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0n)) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a) (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\exists V1n \in ty\_2Enum\_2Enum.((V0w = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V1n)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1b \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lo}\ A\_27a)\ V0a)\ V1b)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V0a))\ (ap\ (\mathsf{c\_2Ewords\_2Ew2n}\ A\_27a)\ V1b)))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in ty\_2Enum\_2Enum.(\forall V1b \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lt}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0a))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V1b))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ 2\ 2)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ 2\ (2^2))\ (\lambda V2sa \in 2.(\lambda V3sb \in 2.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ V2sa)\ V3sb))\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0a)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1b)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ V2sa)\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ V3sb))))))\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EBIT}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0a)))\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EBIT}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V1b)))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0a \in ty\_2Enum\_2Enum.(\ \forall V1b \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_le}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0a))\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V1b))){\Leftrightarrow}(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ 2\ 2)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ 2\ (2^2))\ (\lambda V2sa \in 2.(\lambda V3sb \in 2.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ 2)\ V2sa)\ V3sb))\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0a)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1b)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ V2sa)\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ V3sb))))))\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EBIT}\ (ap\ (\ ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V0a)))\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EBIT}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ V1b)))))) \tag{68}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V1y \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_le}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V0x)){\wedge}(p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_le}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V1y))){\Rightarrow}((p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lt}\ A\_27a)\ V0x)\ V1y)){\Leftrightarrow}(p\ (ap\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_lo}\ A\_27a)\ V0x)\ V1y))))))$$