thm_2Ewords_2EWORD__L_NEG
(TMctaUo6rbt7stNk2Ldcsz8WdKiKPz2QYXf)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\text{-}2Enum\_2Enum}) \tag{6}$$

Definition 10 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\text{-}2Enum\_2Enum}) \tag{7}$$

Let $ty\_2Ebool\_2Eitself : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{8}$$

Let $c\_2Ewords\_2EINT\_\_MIN : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ewords\_2EINT\_\_MIN\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\text{-}2Ebool\_2Eitself\ A\_27}} \tag{9}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (\\ ty\_2Ebool\_2Eitself\ A\_27a) \tag{10}$$

Definition 11 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 12 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 13 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\text{-}2Enum\_2Enum}) \tag{11}$$

Definition 14 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\text{-}2Enum\_2Enum}) \tag{12}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\text{-}2Enum\_2Enum})^{ty\text{-}2Enum\_2Enum}) \tag{13}$$

Definition 15 *We define* c_2Ebit_2EMOD_2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 16 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 17 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Efcp\_2Efinite\_\_image \ A0) \qquad (14)$$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Efcp\_2Edimindex \ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself \ A\_27a)})$$
$$(15)$$

Definition 18 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p \ (ap \ P \ x))$ **then** $(the \ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 19 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ V0P \ (ap \ (\text{c\_2Emin\_2E\_40}$

Definition 20 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 21 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap \ (ap \ \text{c\_2Ebool\_2E\_2F\_5C}$

Definition 22 *We define* c_2Efcp_2Efinite_index *to be* $\lambda A\_27a : \iota.(ap \ (\text{c\_2Emin\_2E\_40} \ (A\_27a^{ty\_2Enum\_2Enu}$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty\_2Efcp\_2Ecart$$
$$A0 \ A1)$$
$$(16)$$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart$$
$$A\_27a \ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image \ A\_27b)})^{(ty\_2Efcp\_2Ecart \ A\_27a \ A\_27b)})$$
$$(17)$$

Definition 23 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart \ A\_27$

Definition 24 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 25 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap \ (\text{c\_2Efcp\_2EFC}$

Definition 26 *We define* c_2Ewords_2Eword__L *to be* $\lambda A\_27a : \iota.(ap \ (\text{c\_2Ewords\_2En2w} \ A\_27a) \ (ap \ (\text{c\_2Ewo}$

Definition 27 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart \ 2 \ A\_27a).(ap$

Assume the following.

$$(\forall V0b \in ty\_2Enum\_2Enum.(p \ (ap \ (ap \ \text{c\_2Ebit\_2EBIT} \ V0b) \ (ap$$
$$(ap \ c\_2Earithmetic\_2EEXP \ (ap \ \text{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \text{c\_2Earithmetic\_2EBIT2}$$
$$\text{c\_2Earithmetic\_2EZERO}))) \ V0b))))$$
$$(18)$$

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Assume the following.

$$True \qquad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow$$
$$(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\qquad (20)$$
$$p\ V0t))))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Ewords\_2EINT\_\_MIN\ A\_27a)$$
$$(c\_2Ebool\_2Ethe\_\_value\ A\_27a)) = (ap\ (ap\ c\_2Earithmetic\_2EEXP$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ ($$
$$c\_2Ebool\_2Ethe\_\_value\ A\_27a)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))$$

$$(21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.($$
$$(p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)\ (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)$$
$$V0n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EBIT}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ ($$
$$ap\ (c\_2Efcp\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a)))$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))$$
$$V0n))))$$

$$(22)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Ewords\_2Eword\_\_msb}\ A\_27a)$$
$$(\mathsf{c\_2Ewords\_2Eword\_\_L}\ A\_27a)))$$