

thm\_2Ewords\_2EWORD\_\_MOD\_\_1  
(TMQFpBGRs8SNHjjMqqx8oziRjFKoC7zA6Fs)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))\ (\lambda V3P \in 2.V3P))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))\ (\lambda V3x \in 2.V3x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define `c_2Earithmetic_2EBIT2` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT2) V0n)$ .

**Definition 8** We define `c_2Earithmetic_2ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let `c_2Earithmetic_2EEXP` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (7)$$

Let `c_2Earithmetic_2EMOD` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 9** We define `c_2Ebit_2EMOD_2EXP` to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0x$ .

Let `c_2Earithmetic_2E_2D` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 10** We define `c_2Ebit_2ESLICE` to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.V0h$ .

Let `c_2Earithmetic_2EDIV` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (10)$$

**Definition 11** We define `c_2Ebit_2EDIV_2EXP` to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V0x$ .

**Definition 12** We define `c_2Ebit_2EBITS` to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.V0h$ .

Let `c_2Earithmetic_2EEVEN` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (11)$$

Let `c_2Earithmetic_2EODD` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 13** We define `c_2Ebool_2EF` to be  $(ap (c\_2Ebool\_2E\_21) 2) (\lambda V0t \in 2.V0t)$ .

**Definition 14** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 15** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E) V0t) c\_2Ebool\_2E\_21)$ .

**Definition 16** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) V0t1) V1t2))$ .

**Definition 17** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \Rightarrow P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 18** We define `c_2Ebool_2E_3F` to be  $\lambda A.\lambda P \in 2^A.\lambda V0P \in 2^A.(ap V0P (ap (c\_2Emin\_2E\_40) V0P))$ .

**Definition 19** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 20** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 21** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2)))$

**Definition 22** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 23** We define  $c\_2Enumeral\_2EiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2ESUC (ap$

**Definition 24** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 25** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

**Definition 26** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_2Enumeral\_2Eexp\_help : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Eexp\_help \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 27** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 28** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (15)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (16)$$

**Definition 29** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Esum\_num\_2EGSUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2EGSUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Epair\_2Eprod ty\_2Enum\_2Enum}) \quad (17)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (18)$$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (19)$$

Let  $ty\_2Efcf\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcf\_2Efinite\_image\ A0) \quad (20)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (21)$$

Let  $c\_2Efcf\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcf\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (22)$$

**Definition 30** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C$

**Definition 31** We define  $c\_2Efcf\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enum}$

Let  $ty\_2Efcf\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcf\_2Ecart\ A0\ A1) \quad (23)$$

Let  $c\_2Efcf\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcf\_2Edest\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcf\_2Efinite\_image\ A\_27b)})^{(ty\_2Efcf\_2Ecart\ A\_27a\ A\_27b)}) \quad (24)$$

**Definition 32** We define  $c\_2Efcf\_2Efcf\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcf\_2Ecart\ A\_27a$

**Definition 33** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebo$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (25)$$

**Definition 34** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap$

**Definition 35** We define  $c\_2Efcf\_2EFCF$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap$

**Definition 36** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcf\_2Ecart\ 2\ A\_27a). (ap\ (ap$

**Definition 37** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. (ap\ (c\_2Efcf\_2EFC$

**Definition 38** We define  $c\_2Ewords\_2Eword\_mod$  to be  $\lambda A\_27a : \iota. \lambda V0v \in (ty\_2Efcf\_2Ecart\ 2\ A\_27a). \lambda V$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2EEXP \\
V0m) c\_2Enum\_2E0) = (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 \\
& c\_2Earithmic\_2EZERO)))) \wedge (\forall V1m \in ty\_2Enum\_2Enum. (\forall V2n \in \\
ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2EEXP V1m) (ap c\_2Enum\_2ESUC \\
V2n)) = (ap (ap c\_2Earithmic\_2E\_2A V1m) (ap (ap c\_2Earithmic\_2EEXP \\
& V1m) V2n))))))
\end{aligned} \tag{26}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m)) \tag{27}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge ((ap ( \\
ap c\_2Earithmic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge ((ap (ap c\_2Earithmic\_2E\_2B \\
& (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmic\_2E\_2B \\
V0m) V1n))) \wedge ((ap (ap c\_2Earithmic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
& V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmic\_2E\_2B V0m) V1n))))))
\end{aligned} \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
(ap (ap c\_2Earithmic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmic\_2E\_2B \\
& V1n) V0m)))) \tag{29}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
(ap (ap c\_2Earithmic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmic\_2E\_2B \\
& V1n) V0m)))) \tag{30}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum. (V0m = (ap c\_2Enum\_2ESUC V1n)))))) \tag{31}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
(p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0n)) (ap \\
c\_2Enum\_2ESUC V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D V0n) \\
& V1m)))))) \tag{32}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( (p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)))))) \quad (33)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)))))))))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V0m))) \quad (38)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \quad (40)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0n))) \quad (41)$$

Assume the following.

$$(\forall V0k \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2EMOD V0k) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = c\_2Enum\_2E0)) \quad (42)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1k) V0n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD V1k) V0n) = V1k)))) \quad (43)$$

Assume the following.

$$(\forall V0b \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0b)) \Rightarrow (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2m \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap (ap c\_2Earithmetic\_2EEXP V0b) V2m)) (ap (ap c\_2Earithmetic\_2EEXP V0b) V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V2m) V1n)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0h \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap (ap (ap c\_2Ebit\_2EBITS V0h) c\_2Enum\_2E0) V1n) = (ap (ap c\_2Earithmetic\_2EMOD V1n) (ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap c\_2Enum\_2ESUC V0h)))))) \quad (45)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1h \in ty\_2Enum\_2Enum. (ap (ap (ap c\_2Ebit\_2ESLICE V1h) c\_2Enum\_2E0) V0n) = (ap (ap (ap c\_2Ebit\_2EBITS V1h) c\_2Enum\_2E0) V0n)))) \quad (46)$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2l \in ty\_2Enum\_2Enum. (\forall V3n \in ty\_2Enum\_2Enum. ( ( \\
& \quad (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1m)) V0h)) \wedge \\
& (p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2l) V1m))) \Rightarrow ((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap (ap (ap c\_2Ebit\_2ESLICE V0h) (ap c\_2Enum\_2ESUC V1m)) V3n)) ( \\
& \quad ap (ap (ap c\_2Ebit\_2ESLICE V1m) V2l) V3n)) = (ap (ap (ap c\_2Ebit\_2ESLICE \\
& \quad V0h) V2l) V3n))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Ebit\_2ESBIT (ap (ap c\_2Ebit\_2EBIT V0x) V1n)) V0x) = (ap \\
& \quad (ap (ap c\_2Ebit\_2ESLICE V0x) V0x) V1n)))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2a \in ty\_2Enum\_2Enum. (\forall V3b \in ty\_2Enum\_2Enum. ( ( \\
& \quad \forall V4x \in ty\_2Enum\_2Enum. ( ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad V1l) V4x)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V4x) V0h))) \Rightarrow ((p \\
& (ap (ap c\_2Ebit\_2EBIT V4x) V2a)) \Leftrightarrow (p (ap (ap c\_2Ebit\_2EBIT V4x) V3b)))))) \Leftrightarrow \\
& ((ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) V2a) = (ap (ap (ap c\_2Ebit\_2EBITS \\
& \quad V0h) V1l) V3b))))))
\end{aligned} \tag{49}$$

Assume the following.

$$True \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. ( ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ( ((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ( (p V0t1) \Leftrightarrow (p V1t2) ) ) ) ) )
\end{aligned} \tag{51}$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2. ( (\forall V1x \in \\
A\_27a. (p V0t) \Leftrightarrow (p V0t) ) ) ) \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. ( ((p V0t1) \wedge \\
& ((p V1t2) \wedge (p V2t3)) \Leftrightarrow ( ((p V0t1) \wedge (p V1t2)) \wedge (p V2t3) ) ) ) ) )
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0t \in 2. ( ((p V0t) \Rightarrow False) \Rightarrow (\neg (p V0t)) ) ) \tag{54}$$

Assume the following.

$$(\forall V0t \in 2. ( (\neg (p V0t)) \Rightarrow ( (p V0t) \Rightarrow False ) ) ) \tag{55}$$



Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (56)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\neg(\forall V1x \in \\
& A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in \\
& (2^{A-27a}).((\forall V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow \\
& ((\forall V3x \in A.27a.(p \ (ap \ V0P \ V3x))) \wedge (\forall V4x \in A.27a.(p \ ( \\
& ap \ V1Q \ V4x)))))) \quad (64)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in 2.(((\forall V2x \in A.27a.(p (ap V0P V2x))) \wedge (p V1Q))) \Leftrightarrow (\forall V3x \in A.27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (65)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (66)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A-27a}).(\forall V1Q \in 2.(((\exists V2x \in A.27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A.27a.((p (ap V0P V3x)) \vee (p V1Q)))))) \quad (67)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A-27a}).(((\exists V2x \in A.27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (68)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A-27a}).(((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V1P V3x)) \vee (p V0Q)))))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (73)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (74)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (75)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2)))))) \quad (76)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27}))) \quad (77)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0P \in ((2^{A_{.27b}})^{A_{.27a}}).((\forall V1x \in A_{.27a}.(\exists V2y \in A_{.27b}.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{.27b}^{A_{.27a}}).(\forall V4x \in A_{.27a}.(p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (78)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in (ty\_2EfcP\_2Ecart A_{.27a} A_{.27b}).(\forall V1y \in (ty\_2EfcP\_2Ecart A_{.27a} A_{.27b}).((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V2i) (ap (c\_2EfcP\_2Edimindex A_{.27b}) (c\_2Ebool\_2Ethe\_value A_{.27b})))) \Rightarrow ((ap (ap (c\_2EfcP\_2EfcP\_index A_{.27a} A_{.27b}) V0x) V2i) = (ap (ap (c\_2EfcP\_2EfcP\_index A_{.27a} A_{.27b}) V1y) V2i)))))) \quad (79)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0g \in (A_{.27a}^{ty\_2Enum\_2Enum}).(\forall V1i \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2EfcP\_2Edimindex A_{.27b}) (c\_2Ebool\_2Ethe\_value A_{.27b})))) \Rightarrow ((ap (ap (c\_2EfcP\_2EfcP\_index A_{.27a} A_{.27b}) (ap (c\_2EfcP\_2EFCP A_{.27a} A_{.27b}) V0g)) V1i) = (ap V0g V1i)))))) \quad (80)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \quad (81)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m)) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2EEXP \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& c\_2Earithmetic\_2EZERO))) \wedge (((ap (ap c\_2Earithmetic\_2EEXP (ap \\
& c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V0n))) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 V0n))) c\_2Earithmetic\_2EZERO))) \wedge \\
& (((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT2 V0n))) = (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap (ap c\_2Enumeral\_2Eexp\_help (ap c\_2Earithmetic\_2EBIT1 V0n)) \\
& c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{85}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{86}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{87}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{89}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (90)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (91)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (93)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (94)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (95)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (96)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (97)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (98)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (99)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (100)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \quad \forall V2f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}). (\forall V3g \in ( \\ & \quad ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}). ((\forall V4x \in ty\_2Enum\_2Enum. \\ & \quad (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0p) V4x)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ & \quad V4x) (ap (ap c\_2Earithmetic\_2E\_2B V0p) V1n)))) \Rightarrow ((ap V2f V4x) = ( \\ & \quad ap V3g V4x)))) \Rightarrow ((ap (ap c\_2Esum\_num\_2EGSUM (ap (ap (c\_2Epair\_2E\_2C \\ & \quad ty\_2Enum\_2Enum ty\_2Enum\_2Enum) V0p) V1n)) V2f) = (ap (ap c\_2Esum\_num\_2EGSUM \\ & \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) V0p) V1n)) \\ & \quad V3g))))))))) \end{aligned} \quad (101)$$

Assume the following.

$$\begin{aligned} & ((\forall V0f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}). ((ap (ap c\_2Esum\_num\_2ESUM \\ & \quad c\_2Enum\_2E0) V0f) = c\_2Enum\_2E0)) \wedge (\forall V1m \in ty\_2Enum\_2Enum. \\ & (\forall V2f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}). ((ap (ap c\_2Esum\_num\_2ESUM \\ & \quad (ap c\_2Enum\_2ESUC V1m)) V2f) = (ap (ap c\_2Earithmetic\_2E\_2B (ap \\ & \quad (ap c\_2Esum\_num\_2ESUM V1m) V2f)) (ap V2f V1m))))))))) \end{aligned} \quad (102)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1f \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}). \\ & ((ap (ap c\_2Esum\_num\_2ESUM V0m) V1f) = (ap (ap c\_2Esum\_num\_2EGSUM \\ & \quad (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) c\_2Enum\_2E0 \\ & \quad V0m)) V1f)))))) \end{aligned} \quad (103)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow ((ap (c\_2Ewords\_2Edimword A\_27a) \\ & \quad (c\_2Ebool\_2Ethe\_value A\_27a)) = (ap (ap c\_2Earithmetic\_2EEXP \\ & \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\ & \quad (ap (c\_2EfcP\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)))) \end{aligned} \quad (104)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\ & \quad (ap (c\_2EfcP\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)))) \end{aligned} \quad (105)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\exists V0m \in ty\_2Enum\_2Enum. ( \\ & \quad (ap (c\_2EfcP\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)) = \\ & \quad (ap c\_2Enum\_2ESUC V0m))) \end{aligned} \quad (106)$$



**Theorem 1**

$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V_0 m \in (\text{ty\_2EfcP\_2Ecart}$   
2  $A_{27a}). ((\text{ap } (\text{ap } (\text{c\_2Ewords\_2Eword\_mod } A_{27a}) V_0 m) (\text{ap } (\text{c\_2Ewords\_2En2w}$   
 $A_{27a}) (\text{ap } \text{c\_2Earithmetic\_2ENUMERAL } (\text{ap } \text{c\_2Earithmetic\_2EBIT1}$   
 $\text{c\_2Earithmetic\_2EZERO})))) = (\text{ap } (\text{c\_2Ewords\_2En2w } A_{27a}) \text{c\_2Enum\_2E0})))$