

thm_2Ewords_2EWORD__NOR__NOT__OR
(TMVyoCN1YS5UnXNqQ3Fm65nCFRbZuTuiHQ)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (1)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (2)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (3)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2EfcP_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (4)$$

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (5)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (6)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (7)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge p\ x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a))))$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Let $ty_2Efcf_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcf_2Efinite_image\ A0) \quad (8)$$

Definition 12 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ c_2Ebool_2E_2F_5C\ A_27a\ P))))$

Definition 13 We define $c_2Efcf_2Efinite_index$ to be $\lambda A_27a : \iota. (ap\ (c_2Emin_2E_40\ A_27a\ (ty_2Enum_2Enum)))$

Let $ty_2Efcf_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcf_2Ecart\ A0\ A1) \quad (9)$$

Let $c_2Efcf_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Efcf_2Edest_cart\ A_27a\ A_27b \in ((A_27a\ (ty_2Efcf_2Efinite_image\ A_27b))\ (ty_2Efcf_2Ecart\ A_27a\ A_27b)) \quad (10)$$

Definition 14 We define $c_2Efcf_2Efcf_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efcf_2Ecart\ A_27a\ A_27b).$

Definition 15 We define c_2Efcf_2EFCF to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a\ ty_2Enum_2Enum). (ap\ (c_2Emin_2E_40\ A_27a\ (ty_2Enum_2Enum))))$

Definition 16 We define $c_2Ewords_2Eword_1comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efcf_2Ecart\ 2\ A_27a).$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (c_2Emin_2E_40\ A_27a\ t2)))))$

Definition 18 We define $c_2Ewords_2Eword_or$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efcf_2Ecart\ 2\ A_27a). \lambda V1w \in (ty_2Efcf_2Ecart\ 2\ A_27a).$

Definition 19 We define $c_2Ewords_2Eword_nor$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_2Efcf_2Ecart\ 2\ A_27a). \lambda V1w \in (ty_2Efcf_2Ecart\ 2\ A_27a).$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ & p\ V0A) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B))))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Efc_2Ecart\ A_27a\ A_27b). (\forall V1y \in (ty_2Efc_2Ecart \\ & A_27a\ A_27b). ((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_2Enum_2Enum. ((p\ (ap \\ & (ap\ c_2Eprim_rec_2E_3C\ V2i)\ (ap\ (c_2Efc_2Edimindex\ A_27b)\ (\\ & c_2Ebool_2Ethe_value\ A_27b)))) \Rightarrow ((ap\ (ap\ (c_2Efc_2Efc_index \\ & A_27a\ A_27b)\ V0x)\ V2i) = (ap\ (ap\ (c_2Efc_2Efc_index\ A_27a\ A_27b) \\ & V1y)\ V2i)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0g \in (A_27a^{ty_2Enum_2Enum}). (\forall V1i \in ty_2Enum_2Enum. \\
& \quad ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1i)\ (ap\ (c_2Efc_2Edimindex\ A_27b) \\
& \quad (c_2Ebool_2Ethe_value\ A_27b)))) \Rightarrow ((ap\ (ap\ (c_2Efc_2Efc_index \\
& \quad A_27a\ A_27b)\ (ap\ (c_2Efc_2EFCP\ A_27a\ A_27b)\ V0g))\ V1i) = (ap\ V0g \\
& \quad V1i))))))
\end{aligned} \tag{21}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Efc_2Ecart \\
& \quad 2\ A_27a). (\forall V1b \in (ty_2Efc_2Ecart\ 2\ A_27a). ((ap\ (ap\ (c_2Ewords_2Eword_nor \\
& \quad A_27a)\ V0a)\ V1b) = (ap\ (c_2Ewords_2Eword_1comp\ A_27a)\ (ap\ (ap\ (\\
& \quad c_2Ewords_2Eword_or\ A_27a)\ V0a)\ V1b))))))
\end{aligned}$$