

thm\_2Ewords\_2EWORD\_\_NOT\_LESS  
 (TMF8xSqX9HGGtBGc8tZPTNvGHnvM5t9vhea)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in omega \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega}) \quad (6)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (7)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B n))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 9** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2EEEXP (V0x * V1n))$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (10)$$

**Definition 10** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2EMOD (V0x / V1n))$

**Definition 11** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V2m \in ty\_2Enum\_2Enum.(c\_2EMOD (V0h * V1l))$

**Definition 12** We define  $c\_2Ebit\_2ESLICE$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V2m \in ty\_2Enum\_2Enum.(c\_2EMOD (V0h / V1l))$

**Definition 13** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x * V1y))$

**Definition 14** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(c\_2EK (A\_27a * A\_27b)))$

**Definition 15** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2ES A\_27a) (A\_27a^{A\_27a})) A\_27a)$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 16** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t))$ .

**Definition 17** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 18** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 19** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c\_2Ebool\_2E\_21 2))(\lambda V2t \in$

**Definition 20** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p(ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p(x))) \text{ else } \perp$

**Definition 21** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap(c\_2Emin\_2E\_40$

**Definition 22** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 23** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 24** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c\_2Ebool\_2E\_21 2))(\lambda V2t \in$

**Definition 25** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 26** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2ESUC (ap$

**Definition 27** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 28** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 29** We define  $c\_2Enumeral\_2EiDUB$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.(ap (ap (ap c\_2Earithmetic\_2EiZ$

Let  $c\_2Enumeral\_2EiSUB : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2EiSUB \in (((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^2) \quad (14)$$

**Definition 30** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(\lambda$

**Definition 31** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_2Enumeral\_2Etexp\_help : \iota$  be given. Assume the following.

$$c\_2Enumeral\_2Etexp\_help \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty\_2Ebool\_2Eitself A0) \quad (16)$$

Let  $c\_2Ewords\_2EINT\_MIN : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Ewords\_2EINT\_MIN A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)})$$

(17)

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_0.\text{nonempty } A_0 \Rightarrow & \forall A_1.\text{nonempty } A_1 \Rightarrow \text{nonempty } (ty\_2Epair\_2Eprod \\ & A_0 A_1) \end{aligned} \quad (18)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (19)$$

**Definition 32** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2$

Let  $ty\_2Efcp\_2Efinit\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_0.\text{nonempty } A_0 \Rightarrow \text{nonempty } (ty\_2Efcp\_2Efinit\_image A_0) \quad (20)$$

Let  $c\_2Ebool\_2Eth\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & c\_2Ebool\_2Eth\_value A\_27a \in ( \\ & ty\_2Ebool\_2Eitself A\_27a) \end{aligned} \quad (21)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Efcp\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (22)$$

**Definition 33** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap c\_2Ebool\_2E\_2F\_5C$

**Definition 34** We define  $c\_2Efcp\_2Efinit\_index$  to be  $\lambda A\_27a : \iota. (ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_0.\text{nonempty } A_0 \Rightarrow & \forall A_1.\text{nonempty } A_1 \Rightarrow \text{nonempty } (ty\_2Efcp\_2Ecart \\ & A_0 A_1) \end{aligned} \quad (23)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart \\ & A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinit\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \end{aligned} \quad (24)$$

**Definition 35** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a$

**Definition 36** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (25)$$

**Definition 37** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c\_2Ewords\_2Ew2n\ A\_27a)\ V0w)$ .  
 Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (26)$$

**Definition 38** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2EBIT1\ A\_27a)\ V0n)$ .

**Definition 39** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.(ap\ (c\_2Ebit\_2EBIT\ A\_27a)\ V0b)$ .

**Definition 40** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ (c\_2Efcp\_2EFCP\ A\_27a)\ V0g))$ .

**Definition 41** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Ewords\_2En2w\ A\_27a)\ V0n)$ .

**Definition 42** We define  $c\_2Ewords\_2Eword\_2comp$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ V0w)$ .

**Definition 43** We define  $c\_2Ewords\_2Eword\_msb$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (c\_2Ewords\_2Eword\_msb\ A\_27a)\ V0w)$ .

**Definition 44** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27b. (ap\ (c\_2Ebool\_2ELET\ A\_27a)\ V0f)))$ .

**Definition 45** We define  $c\_2Ewords\_2Enzcv$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). \lambda V1b \in (ty\_2Efcp\_2Ecart\ 2\ A\_27b). (ap\ (c\_2Ewords\_2Enzcv\ A\_27a)\ V0a)$ .

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (27)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (28)$$

**Definition 46** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c^{A\_27a})^{A\_27b}). (ap\ (c\_2Epair\_2EUNCURRY\ A\_27a)\ V0f)$ .

**Definition 47** We define  $c\_2Ewords\_2Eword\_lt$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). \lambda V1b \in (ty\_2Efcp\_2Ecart\ 2\ A\_27b). (ap\ (c\_2Ewords\_2Eword\_lt\ A\_27a)\ V0a)$ .

**Definition 48** We define  $c\_2Ewords\_2Eword\_le$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). \lambda V1b \in (ty\_2Efcp\_2Ecart\ 2\ A\_27b). (ap\ (c\_2Ewords\_2Eword\_le\ A\_27a)\ V0a)$ .

**Definition 49** We define  $c\_2Ewords\_2Eword\_add$  to be  $\lambda A\_27a : \iota. \lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). \lambda V1w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27b). (ap\ (c\_2Ewords\_2Eword\_add\ A\_27a)\ V0v)$ .

**Definition 50** We define  $c\_2Ewords\_2Eword\_sub$  to be  $\lambda A\_27a : \iota. \lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). \lambda V1w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27b). (ap\ (c\_2Ewords\_2Eword\_sub\ A\_27a)\ V0v)$ .

**Definition 51** We define  $c\_2Ewords\_2Eword\_L$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Ewords\_2En2w\ A\_27a) (ap\ (c\_2Ewords\_2En2w\ A\_27a) V0a))$ .

Assume the following.

$$\begin{aligned} ((ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)) = \\ (ap\ c\_2Enum\_2ESUC\ c\_2Enum\_2E0)) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ c\_2Enum\_2E0) = V0m)) \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Enum\_2E0)\ V0m) = V0m) \wedge (((ap\ ( \\ ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ c\_2Enum\_2E0) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \\ (ap\ c\_2Enum\_2ESUC\ V0m))\ V1n) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ c\_2Enum\_2ESUC\ \\ V1n)) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \\ V1n)\ V0m)))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ \forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m) \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p)))))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ \\ c\_2Enum\_2E0)\ V0n))) \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ \\ V1n)\ V0m)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \\ c\_2Enum\_2E0)\ V0m) = c\_2Enum\_2E0) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \\ V0m)\ c\_2Enum\_2E0) = V0m))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic_2E_2D V0m) V1n) = c_2Enum_2E0) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0m)) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m))) \quad (38)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m))) \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A V1n) V2p))))))) \end{aligned} \quad (41)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V0m))) \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p ( \\ & ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & (ap c\_2Enum\_2ESUC V1n)) V0m))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\ & V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\ & V1n)) V0m))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\ & c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO))) V0n))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & \forall V2p \in ty\_2Enum\_2Enum. ((V0m = (ap (ap c\_2Earithmetic\_2E\_2D \\ & V1n) V2p)) \Leftrightarrow (((ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p) = V1n) \vee ((p \\ & (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) c\_2Enum\_2E0)) \wedge (p (ap (ap \\ & c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1r \in ty\_2Enum\_2Enum. \\
 & (p (ap (ap c_2Eprim_rec_2E_3C V1r) V0n)) \Rightarrow (\forall V2q \in ty\_2Enum\_2Enum. \\
 & ((ap (ap c_2Earithmetic_2EDIV (ap (ap c_2Earithmetic_2E_2B (ap \\
 & (ap c_2Earithmetic_2E_2A V2q) V0n)) V1r)) V0n) = V2q)))) \\
 \end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. \\
 & (p (ap (ap c_2Eprim_rec_2E_3C V1k) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD \\
 & V1k) V0n) = V1k)))) \\
 \end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD c_2Enum_2E0) \\
 & V0n) = c_2Enum_2E0))) \\
 \end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
 & c_2Enum_2E0) V0n)) \Rightarrow (((ap (ap c_2Earithmetic_2EDIV V0n) V0n) = \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge \\
 & ((ap (ap c_2Earithmetic_2EMOD V0n) V0n) = c_2Enum_2E0)))) \\
 \end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (\forall V1a \in ty\_2Enum\_2Enum. \\
 & (\forall V2b \in ty\_2Enum\_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\
 & V1a) V2b)) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B \\
 & V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\
 & V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))) \\
 \end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c_2Earithmetic_2EEEXP \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
 & V0n) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & c_2Earithmetic_2EZERO))) \wedge ((ap (ap c_2Earithmetic_2EEEXP V0n) \\
 & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
 & V0n))) \\
 \end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0a \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic_2E_2D \\
 & (ap c_2Enum_2ESUC V0a)) V0a) = (ap c_2Earithmetic_2ENUMERAL (ap \\
 & c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
 \end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0r \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0r) V1n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EDIV \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V1n) V0r)) V1n) = (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))) \\
 \end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\
 & (ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
 & c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V0n)))) \\
 \end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
 & \forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ebit\_2EBITS V0h) V1l) \\
 & V2n) = (ap (ap c\_2Earithmetic\_2EMOD (ap (ap c\_2Earithmetic\_2EDIV \\
 & V2n) (ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V1l))) ( \\
 & ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL (ap \\
 & c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap (ap c\_2Earithmetic\_2E\_2D \\
 & (ap c\_2Enum\_2ESUC V0h)) V1l))))))) \\
 \end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
 & \forall V2n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap \\
 & (ap (ap c\_2Ebit\_2EBITS V0h) V1l) V2n)) (ap (ap c\_2Earithmetic\_2EEEXP \\
 & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) \\
 & (ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Enum\_2ESUC V0h)) V1l))))))) \\
 \end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1l \in ty\_2Enum\_2Enum. ( \\
 & (ap (ap (ap c\_2Ebit\_2EBITS V0h) V1l) c\_2Enum\_2E0) = c\_2Enum\_2E0))) \\
 \end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & (ap (ap (ap c\_2Ebit\_2EBITS V0h) c\_2Enum\_2E0) V1n) = (ap (ap c\_2Earithmetic\_2EMOD \\
 & V1n) (ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL \\
 & (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) (ap c\_2Enum\_2ESUC \\
 & V0h))))))) \\
 \end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1h \in ty\_2Enum\_2Enum. (ap (ap (ap c\_2Ebit\_2ESLICE V1h) c\_2Enum\_2E0) V0n) = (ap (ap (ap c\_2Ebit\_2EBITS V1h) c\_2Enum\_2E0) V0n)))) \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0h \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \forall V2l \in ty\_2Enum\_2Enum. (\forall V3n \in ty\_2Enum\_2Enum. (( p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1m)) V0h)) \wedge \\ & (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2l) V1m))) \Rightarrow ((ap (ap c\_2Earithmetic\_2E\_2B (ap (ap (ap c\_2Ebit\_2ESLICE V0h) (ap c\_2Enum\_2ESUC V1m)) V3n)) ( ap (ap (ap c\_2Ebit\_2ESLICE V1m) V2l) V3n)) = (ap (ap (ap c\_2Ebit\_2ESLICE V0h) V2l) V3n))))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1a \in ty\_2Enum\_2Enum. ( \neg((ap (ap (ap c\_2Ebit\_2EBITS V0n) V0n) V1a) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \Leftrightarrow ((ap (ap c\_2Ebit\_2EBITS V0n) V0n) V1a) = c\_2Enum\_2E0)))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0b \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( (p (ap (ap c\_2Ebit\_2EBIT V0b) V1n)) \Leftrightarrow ((ap (ap (ap c\_2Ebit\_2ESLICE V0b) V1n) = (ap (ap c\_2Earithmetic\_2EEEXP (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)) V0b))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & (\forall V0b \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \neg(p (ap (ap c\_2Ebit\_2EBIT V0b) V1n))) \Leftrightarrow ((ap (ap (ap c\_2Ebit\_2ESLICE V0b) V0b) V1n) = c\_2Enum\_2E0)))) \end{aligned} \quad (67)$$

Assume the following.

$$True \quad (68)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (70)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (71)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow \forall A_{27b}. nonempty A_{27b} \Rightarrow \\ & \forall V0f \in (A_{27b}^{A_{27a}}). (\forall V1x \in A_{27a}. ((ap (ap (c_2Ebool_2ELT \\ & A_{27a} A_{27b}) V0f) V1x) = (ap V0f V1x)))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A_{27a}. (p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (73)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\ ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))))) \quad (74)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (75)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (76)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t))))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow \\ & (p V0t)) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (79)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True)))) \quad (80)$$

Assume the following.

$$\forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}. (V0x = V0x)) \quad (81)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0x \in A_{\text{27a}}. ((V0x = V0x) \Leftrightarrow \text{True})) \quad (82)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0x \in A_{\text{27a}}. (\forall V1y \in A_{\text{27a}}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (83)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p \ V0t))))))) \quad (84)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0t1 \in A_{\text{27a}}. (\forall V1t2 \in A_{\text{27a}}. (((ap \ (ap \ (ap \ (c_{\text{2Ebool\_2ECOND}} \ A_{\text{27a}}) \ c_{\text{2Ebool\_2ET}}) \ V0t1) \\ & V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_{\text{2Ebool\_2ECOND}} \ A_{\text{27a}}) \ c_{\text{2Ebool\_2EF}}) \\ & V0t1) \ V1t2) = V1t2)))))) \end{aligned} \quad (85)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}}). (((\neg(\forall V1x \in A_{\text{27a}}. (p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_{\text{27a}}. (\neg(p \ (ap \ V0P \ V2x))))))) \quad (86)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p \ V0A) \vee (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (87)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee (p \ V0A)))) \quad (88)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))) \quad (89)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))) \quad (90)$$

Assume the following.

$$(\forall V0t \in 2. (((p \ V0t) \Rightarrow \text{False}) \Leftrightarrow ((p \ V0t) \Leftrightarrow \text{False}))) \quad (91)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (92)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1) \wedge (\neg(p V1t2))))))) \quad (93)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \end{aligned} \quad (94)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2EI \\ A\_27a) V0x) = V0x)) \quad (95)$$

Assume the following.

$$\begin{aligned} & (((ap c\_2Enum\_2ESUC c\_2Earithmetic\_2EZERO) = (ap c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap \\ & c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2EBIT1 V0n)) = (ap c\_2Earithmetic\_2EBIT2 \\ & V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2EBIT2 \\ & V1n)) = (ap c\_2Earithmetic\_2EBIT1 (ap c\_2Enum\_2ESUC V1n))))))) \end{aligned} \quad (96)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& ((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
(ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\
c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. \\
& (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Enum\_2ESUC V17n)))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
(ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& ((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((c\_2Earithmetic\_2ZERO = (ap c\_2Earithmetic\_2EBIT1 V0n)) \Leftrightarrow False) \wedge \\
& (((ap c\_2Earithmetic\_2EBIT1 V0n) = c\_2Earithmetic\_2ZERO) \Leftrightarrow \\
& False) \wedge (((c\_2Earithmetic\_2ZERO = (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap c\_2Earithmetic\_2EBIT2 V0n) = c\_2Earithmetic\_2ZERO) \Leftrightarrow \\
& False) \wedge (((ap c\_2Earithmetic\_2EBIT1 V0n) = (ap c\_2Earithmetic\_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c\_2Earithmetic\_2EBIT2 V0n) = (ap c\_2Earithmetic\_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c\_2Earithmetic\_2EBIT1 V0n) = (ap c\_2Earithmetic\_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c\_2Earithmetic\_2EBIT2 V0n) = (ap c\_2Earithmetic\_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m)))))))))) \\
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2ZERO) (ap c\_2Earithmetic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2ZERO) \\
& (ap c\_2Earithmetic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmetic\_2ZERO) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m)))))))))) \\
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2ZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2ZERO) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2ZERO) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) \\
& (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m)))))))))) \\
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Earithmetic_2ZERO) = c_2Earithmetic_2ZERO) \wedge \\
& (((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2ZERO)) = \\
& \quad c_2Earithmetic_2ZERO) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\
& \quad c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad V0n))) = (ap c_2Earithmetic_2EBIT2 (ap c_2Eprim_rec_2EPRE (ap \\
& \quad c_2Earithmetic_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT2 \\
& \quad V1n))) = (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\
& \quad V1n)))) \wedge (\forall V2n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2EBIT2 V2n)) = (ap c_2Earithmetic_2EBIT1 V2n))))))) \\
& \tag{102}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1b \in 2. (\forall V2n \in ty\_2Enum\_2Enum. \\
& (\forall V3m \in ty\_2Enum\_2Enum. (((ap (ap (ap c\_2Enumeral\_2EiSUB \\
& V1b) c\_2Earithmetic\_2EZERO) V0x) = c\_2Earithmetic\_2EZERO) \wedge \\
& ((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) c\_2Earithmetic\_2EZERO) = \\
& V2n) \wedge (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) c\_2Earithmetic\_2EZERO) = (ap c\_2Enumeral\_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT1 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) c\_2Earithmetic\_2EZERO) = (ap c\_2Earithmetic\_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT1 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Enumeral\_2EiDUB \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) (ap c\_2Earithmetic\_2EBIT2 \\
& V2n)) (ap c\_2Earithmetic\_2EBIT2 V3m)) = (ap c\_2Earithmetic\_2EBIT1 \\
& (ap (ap (ap c\_2Enumeral\_2EiSUB c\_2Ebool\_2EF) V2n) V3m))))))))))))))) \\
& (103)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V0n) \\
& V1m)) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V1m) V0n)) (ap c\_2Earithmetic\_2ENUMERAL (ap (ap (ap c\_2Enumeral\_2EiSUB \\
& c\_2Ebool\_2ET) V0n) V1m))) c\_2Enum\_2E0)))) \\
& (104)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0acc \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (((ap (ap c_2Enumeral_2Etxp_help c_2Earithmetic_2ZERO) V0acc) = \\
& (ap c_2Earithmetic_2EBIT2 V0acc)) \wedge ((ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT1 V1n)) V0acc) = (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 V1n))) (ap \\
& c_2Earithmetic_2EBIT1 V0acc))) \wedge ((ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT2 V1n)) V0acc) = (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT1 V1n)) (ap c_2Earithmetic_2EBIT1 V0acc)))))) \\
& (105)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap (ap c_2Earithmetic_2EXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2ZERO))) \\
& c_2Enum\_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2ZERO))) \wedge (((ap (ap c_2Earithmetic_2EXP (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2ZERO))) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V0n))) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 V0n))) c_2Earithmetic_2ZERO))) \wedge \\
& ((ap (ap c_2Earithmetic_2EXP (ap c_2Earithmetic_2ENUMERAL ( \\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2ZERO))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n))) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap (ap c_2Enumeral_2Etxp_help (ap c_2Earithmetic_2EBIT1 V0n) \\
& c_2Earithmetic_2ZERO)))))) \\
& (106)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow \forall A\_27c. \\
& nonempty A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\
& A\_27a. (\forall V2y \in A\_27b. ((ap (ap (c_2Epair_2EUNCURRY A\_27a \\
& A\_27b A\_27c) V0f) (ap (ap (c_2Epair_2E2C A\_27a A\_27b) V1x) V2y)) = \\
& (ap (ap V0f V1x) V2y)))))) \\
& (107)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (108)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (109)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (110)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (111)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (112)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (113)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee ((\neg(p V0p)) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (114)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (115)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \wedge ((p V0p) \vee ((\neg(p V2r)) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (116)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (117)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))))) \quad (118)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (119)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (120)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (121)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (122)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & ((ap (c_2Ewords_2Edimword A_{27a}) \\ & (c_2Ebool_2Ethe_value A_{27a})) = (ap (ap c_2Earithmetic_2EEEXP \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\ & (ap (c_2Efcp_2Edimindex A_{27a}) (c_2Ebool_2Ethe_value A_{27a})))) \end{aligned} \quad (123)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & ((ap (c_2Ewords_2EINT_MIN A_{27a}) \\ & (c_2Ebool_2Ethe_value A_{27a})) = (ap (ap c_2Earithmetic_2EEEXP \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\ & (ap (ap c_2Earithmetic_2E_2D (ap (c_2Efcp_2Edimindex A_{27a}) ( \\ & c_2Ebool_2Ethe_value A_{27a}))) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (124)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & (\exists V0m \in ty_2Enum_2Enum. ( \\ & (ap (c_2Efcp_2Edimindex A_{27a}) (c_2Ebool_2Ethe_value A_{27a})) = \\ & (ap c_2Enum_2ESUC V0m))) \end{aligned} \quad (125)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & (\forall V0n \in ty_2Enum_2Enum. ( \\ & (ap (c_2Ewords_2Ew2n A_{27a}) (ap (c_2Ewords_2En2w A_{27a}) V0n)) = \\ & (ap (ap c_2Earithmetic_2EMOD V0n) (ap (c_2Ewords_2Edimword A_{27a}) \\ & (c_2Ebool_2Ethe_value A_{27a})))))) \end{aligned} \quad (126)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & (\forall V0w \in (ty_2Efcp_2Ecart \\ & 2 A_{27a}). ((ap (c_2Ewords_2En2w A_{27a}) (ap (c_2Ewords_2Ew2n A_{27a}) \\ & V0w)) = V0w))) \end{aligned} \quad (127)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & (\forall V0n \in ty_2Enum_2Enum. ( \\ & (ap (c_2Ewords_2En2w A_{27a}) (ap (ap c_2Earithmetic_2EMOD V0n) \\ & (ap (c_2Ewords_2Edimword A_{27a}) (c_2Ebool_2Ethe_value A_{27a})))) = \\ & (ap (c_2Ewords_2En2w A_{27a}) V0n))) \end{aligned} \quad (128)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0m \in ty\_2Enum\_2Enum. ( \\ & \forall V1n \in ty\_2Enum\_2Enum. (((ap (c\_2Ewords\_2En2w\ A_{27a})\ V0m) = \\ & (ap (c\_2Ewords\_2En2w\ A_{27a})\ V1n)) \Leftrightarrow ((ap (ap c\_2Earithmetic\_2EMOD \\ & V0m) (ap (c\_2Ewords\_2Edimword\ A_{27a}) (c\_2Ebool\_2Ethe\_value \\ & A_{27a}))) = (ap (ap c\_2Earithmetic\_2EMOD\ V1n) (ap (c\_2Ewords\_2Edimword \\ & A_{27a}) (c\_2Ebool\_2Ethe\_value\ A_{27a}))))))) \end{aligned} \quad (129)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0w \in (ty\_2Efcp\_2Ecart \\ & 2\ A_{27a}). (\exists V1n \in ty\_2Enum\_2Enum. ((V0w = (ap (c\_2Ewords\_2En2w \\ & A_{27a})\ V1n)) \wedge (p (ap (ap c\_2Eprim\_rec\_2E\_3C\ V1n) (ap (c\_2Ewords\_2Edimword \\ & A_{27a}) (c\_2Ebool\_2Ethe\_value\ A_{27a}))))))) \end{aligned} \quad (130)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0v \in (ty\_2Efcp\_2Ecart \\ & 2\ A_{27a}). (\forall V1w \in (ty\_2Efcp\_2Ecart\ 2\ A_{27a}). (((ap (c\_2Ewords\_2Ew2n \\ & A_{27a})\ V0v) = (ap (c\_2Ewords\_2Ew2n\ A_{27a})\ V1w)) \Leftrightarrow (V0v = V1w)))) \end{aligned} \quad (131)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0m \in ty\_2Enum\_2Enum. ( \\ & \forall V1n \in ty\_2Enum\_2Enum. (((ap (ap (c\_2Ewords\_2Eword\_add \\ & A_{27a}) (ap (c\_2Ewords\_2En2w\ A_{27a})\ V0m)) (ap (c\_2Ewords\_2En2w \\ & A_{27a})\ V1n)) = (ap (c\_2Ewords\_2En2w\ A_{27a}) (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m)\ V1n))))))) \end{aligned} \quad (132)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0n \in ty\_2Enum\_2Enum. ( \\ & (ap (c\_2Ewords\_2Eword\_2comp\ A_{27a}) (ap (c\_2Ewords\_2En2w\ A_{27a}) \\ & V0n)) = (ap (c\_2Ewords\_2En2w\ A_{27a}) (ap (ap c\_2Earithmetic\_2E\_2D \\ & (ap (c\_2Ewords\_2Edimword\ A_{27a}) (c\_2Ebool\_2Ethe\_value\ A_{27a}))) \\ & (ap (ap c\_2Earithmetic\_2EMOD\ V0n) (ap (c\_2Ewords\_2Edimword\ A_{27a}) \\ & (c\_2Ebool\_2Ethe\_value\ A_{27a}))))))) \end{aligned} \quad (133)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0n \in ty\_2Enum\_2Enum. ( \\ & (p (ap (c\_2Ewords\_2Eword\_msb\ A_{27a}) (ap (c\_2Ewords\_2En2w\ A_{27a}) \\ & V0n))) \Leftrightarrow (p (ap (ap c\_2Ebit\_2EBIT (ap (ap c\_2Earithmetic\_2E\_2D \\ & (ap (c\_2Efcp\_2Edimindex\ A_{27a}) (c\_2Ebool\_2Ethe\_value\ A_{27a}))) \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) \\ & V0n))))))) \end{aligned} \quad (134)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0w \in (ty\_2Efcp\_2Ecart \\ & 2\ A_{27a}).((ap\ (ap\ (c\_2Ewords\_2Eword\_sub\ A_{27a})\ V0w)\ V0w) = (ap\ \\ & (c\_2Ewords\_2En2w\ A_{27a})\ c\_2Enum\_2E0))) \end{aligned} \quad (135)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & ((ap\ (c\_2Ewords\_2Eword\_2comp\ \\ & A_{27a})\ (ap\ (c\_2Ewords\_2En2w\ A_{27a})\ c\_2Enum\_2E0)) = (ap\ (c\_2Ewords\_2En2w\ \\ & A_{27a})\ c\_2Enum\_2E0))) \end{aligned} \quad (136)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0v \in (ty\_2Efcp\_2Ecart \\ & 2\ A_{27a}).(\forall V1w \in (ty\_2Efcp\_2Ecart\ 2\ A_{27a}).(\forall V2x \in \\ & (ty\_2Efcp\_2Ecart\ 2\ A_{27a}).(((ap\ (ap\ (c\_2Ewords\_2Eword\_sub\ \\ & A_{27a})\ V0v)\ V1w) = V2x) \Leftrightarrow (V0v = (ap\ (ap\ (c\_2Ewords\_2Eword\_add\ A_{27a})\ \\ & V2x)\ V1w)))))) \end{aligned} \quad (137)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0v \in (ty\_2Efcp\_2Ecart \\ & 2\ A_{27a}).(\forall V1w \in (ty\_2Efcp\_2Ecart\ 2\ A_{27a}).(\forall V2x \in \\ & (ty\_2Efcp\_2Ecart\ 2\ A_{27a}).(((ap\ (ap\ (c\_2Ewords\_2Eword\_sub\ \\ & A_{27a})\ V0v)\ V1w) = (ap\ (ap\ (c\_2Ewords\_2Eword\_sub\ A_{27a})\ V2x)\ V1w)) \Leftrightarrow \\ & (V0v = V2x)))))) \end{aligned} \quad (138)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0w \in (ty\_2Efcp\_2Ecart \\ & 2\ A_{27a}).(\forall V1v \in (ty\_2Efcp\_2Ecart\ 2\ A_{27a}).((ap\ (c\_2Ewords\_2Eword\_2comp\ \\ & A_{27a})\ (ap\ (ap\ (c\_2Ewords\_2Eword\_sub\ A_{27a})\ V1v)\ V0w)) = (ap\ (ap\ \\ & (c\_2Ewords\_2Eword\_sub\ A_{27a})\ V0w)\ V1v)))))) \end{aligned} \quad (139)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0a \in (ty\_2Efcp\_2Ecart \\ & 2\ A_{27a}).(((\neg(V0a = (ap\ (c\_2Ewords\_2En2w\ A_{27a})\ c\_2Enum\_2E0))) \wedge \\ & (\neg(V0a = (c\_2Ewords\_2Eword\_L\ A_{27a})))) \Rightarrow ((\neg(p\ (ap\ (c\_2Ewords\_2Eword\_msb\ \\ & A_{27a})\ V0a)) \Leftrightarrow (p\ (ap\ (c\_2Ewords\_2Eword\_msb\ A_{27a})\ (ap\ (c\_2Ewords\_2Eword\_2comp\ \\ & A_{27a})\ V0a))))))) \end{aligned} \quad (140)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\neg(p\ (ap\ (c\_2Ewords\_2Eword\_msb\ \\ & A_{27a})\ (ap\ (c\_2Ewords\_2En2w\ A_{27a})\ c\_2Enum\_2E0)))) \quad (141)$$

**Theorem 1**

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0a \in (ty\_2Efc_2Ecart \\ 2 A\_27a).(\forall V1b \in (ty\_2Efc_2Ecart 2 A\_27a).((\neg(p (ap (\\ ap (c\_2Ewords\_2Eword\_lt A\_27a) V0a) V1b))) \Leftrightarrow (p (ap (ap (ap (c\_2Ewords\_2Eword\_le \\ A\_27a) V1b) V0a))))))$$