thm_2Ewords_2EWORD__SET__INDUCT (TMGH7N1vh7bx1AUCSW2Aah7tmbFNwB2v2FH)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 10 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a:\iota.\lambda V0x\in A\_27a.\lambda V1s\in(2^{A\_27a}).(ap\ (c\_$

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t\in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 12 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a:\iota.(\lambda V0x\in A_27a.$c_2Ebool_2EF).

Definition 13 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Let $ty\_2Efcp\_2Ecart:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{4}$$

Assume the following.

$$(\forall V0t\in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{5}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{6}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P\in(2^{(2^{A\_27a})}).(((p\ (ap\ V0P\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)))\wedge(\forall V1s\in(2^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s))\wedge(p\ (ap\ V0P\ V1s)))\Rightarrow(\forall V2e\in A\_27a.((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2e)\ V1s)))\Rightarrow(p\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V2e)\ V1s))))))))\Rightarrow(\forall V3s\in(2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V3s))\Rightarrow(p\ (ap\ V0P\ V3s)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s\in(2^{(ty\_2Efcp\_2Ecart\ 2\ A\_27a)}).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ V0s))) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P\in(2^{(2^{(ty\_2Efcp\_2Ecart\ 2\ A\_27a)})}).(((p\ (ap\ V0P\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))))\wedge(\forall V1s\in(2^{(ty\_2Efcp\_2Ecart\ 2\ A\_27a)}).((p\ (ap\ V0P\ V1s))\Rightarrow(\forall V2e\in(ty\_2Efcp\_2Ecart\ 2\ A\_27a).((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ V2e)\ V1s)))\Rightarrow(p\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ V2e)\ V1s))))))))\Rightarrow(\forall V3s\in(2^{(ty\_2Efcp\_2Ecart\ 2\ A\_27a)}).(p\ (ap\ V0P\ V3s)))))$$