

thm_2Ewords_2EWORD__SUB__INTRO
(TMLJ8FjQ2zRSr46mjLH7PyUsMHHhnj1ivno)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}})) (\lambda V1t \in 2.V1t)) (\lambda V2t \in 2.V2t))$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define `c_2Emarker_2E_2AC` to be $\lambda V0b1 \in 2.\lambda V1b2 \in 2.(ap (ap c_2Ebool_2E_2F_5C V0b1) V1b2)$

Definition 7 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 9 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let `ty_2Efc_2Efinite_image` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efc_2Efinite_image A0) \quad (1)$$

Let `ty_2Ebool_2Eitself` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (2)$$

Let `c_2Ebool_2Ethe_value` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow c_2Ebool_2Ethe_value A_{27a} \in (ty_2Ebool_2Eitself A_{27a}) \quad (3)$$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (4)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (5)$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (6)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 12 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 13 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 14 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E_2F_5C$

Definition 15 We define $c_2Efc_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum$

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efc_2Ecart\ A0\ A1) \quad (9)$$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})(ty_2Efc_2Ecart\ A_27a\ A_27b)) \quad (10)$$

Definition 16 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A_27a$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (12)$$

Definition 19 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (ap c_2Ebool_2EBIT2))))))$

Definition 22 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap c_2Ebool_2EBIT2)))$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 23 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap (ap c_2Esum_num_2ESUM))$

Definition 24 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (15)$$

Definition 25 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_2Ebit_2ESBIT))$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (16)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (17)$$

Definition 26 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_2Ebit_2ESBIT))$

Definition 27 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V2n \in ty_2Enum_2Enum.(ap (ap c_2Ebit_2ESBIT))$

Definition 28 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_2Ebit_2ESBIT))$

Definition 29 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap (ap c_2Esum_num_2ESUM)))$

Definition 30 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efc_2EFCP))$

Definition 31 We define $c_Ewords_Eword_add$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_Efc_Ecart\ 2\ A_27a). \lambda V$

Let $c_Ewords_Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_Ewords_Edimword\ A_27a \in (ty_Enum_Enum^{(ty_Ebool_Eitself\ A_27a)}) \quad (18)$$

Definition 32 We define $c_Ewords_Eword_comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).$

Definition 33 We define $c_Ewords_Eword_sub$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_Efc_Ecart\ 2\ A_27a). \lambda V$

Let $c_Earithmetic_E_EA : \iota$ be given. Assume the following.

$$c_Earithmetic_E_EA \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (19)$$

Definition 34 We define $c_Ewords_Eword_mul$ to be $\lambda A_27a : \iota. \lambda V0v \in (ty_Efc_Ecart\ 2\ A_27a). \lambda V$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (21)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (24)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (25)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0v \in (ty.2EfcP.2Ecart \\ & \ 2 \ A.27a).(\forall V1w \in (ty.2EfcP.2Ecart \ 2 \ A.27a).(\forall V2x \in \\ & (ty.2EfcP.2Ecart \ 2 \ A.27a).((ap \ (ap \ (c.2Ewords.2Eword_add \ A.27a) \\ & V0v) \ (ap \ (ap \ (c.2Ewords.2Eword_add \ A.27a) \ V1w) \ V2x)) = (ap \ (ap \ (\\ & \ c.2Ewords.2Eword_add \ A.27a) \ (ap \ (ap \ (c.2Ewords.2Eword_add \\ & \ A.27a) \ V0v) \ V1w)) \ V2x)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0v \in (ty.2EfcP.2Ecart \\ & \ 2 \ A.27a).(\forall V1w \in (ty.2EfcP.2Ecart \ 2 \ A.27a).(\forall V2x \in \\ & (ty.2EfcP.2Ecart \ 2 \ A.27a).((ap \ (ap \ (c.2Ewords.2Eword_mul \ A.27a) \\ & V0v) \ (ap \ (ap \ (c.2Ewords.2Eword_mul \ A.27a) \ V1w) \ V2x)) = (ap \ (ap \ (\\ & \ c.2Ewords.2Eword_mul \ A.27a) \ (ap \ (ap \ (c.2Ewords.2Eword_mul \\ & \ A.27a) \ V0v) \ V1w)) \ V2x)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0v \in (ty.2EfcP.2Ecart \\ & \ 2 \ A.27a).(\forall V1w \in (ty.2EfcP.2Ecart \ 2 \ A.27a).((ap \ (ap \ (c.2Ewords.2Eword_add \\ & \ A.27a) \ V0v) \ V1w) = (ap \ (ap \ (c.2Ewords.2Eword_add \ A.27a) \ V1w) \ V0v)))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0v \in (ty.2EfcP.2Ecart \\ & \ 2 \ A.27a).(\forall V1w \in (ty.2EfcP.2Ecart \ 2 \ A.27a).((ap \ (ap \ (c.2Ewords.2Eword_mul \\ & \ A.27a) \ V0v) \ V1w) = (ap \ (ap \ (c.2Ewords.2Eword_mul \ A.27a) \ V1w) \ V0v)))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0w \in (ty.2EfcP.2Ecart \\ & \ 2 \ A.27a).((ap \ (c.2Ewords.2Eword_2comp \ A.27a) \ (ap \ (c.2Ewords.2Eword_2comp \\ & \ A.27a) \ V0w)) = V0w)) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\ 2\ A.27a).(\forall V1w \in (ty_2EfcP_2Ecart\ 2\ A.27a).((ap\ (ap\ (c_2Ewords_2Eword_sub \\ A.27a)\ (ap\ (c_2Ewords_2Eword_2comp\ A.27a)\ V0v))\ V1w) = (ap\ (c_2Ewords_2Eword_2comp \\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_add\ A.27a)\ V0v)\ V1w)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0v \in (ty_2EfcP_2Ecart \\ 2\ A.27a).(\forall V1w \in (ty_2EfcP_2Ecart\ 2\ A.27a).((ap\ (c_2Ewords_2Eword_2comp \\ A.27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A.27a)\ V0v)\ V1w)) = (ap\ (ap \\ (c_2Ewords_2Eword_mul\ A.27a)\ (ap\ (c_2Ewords_2Eword_2comp \\ A.27a)\ V0v))\ V1w)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty_2EfcP_2Ecart \\ 2\ A.27a).((ap\ (c_2Ewords_2Eword_2comp\ A.27a)\ V0w) = (ap\ (ap\ (\\ c_2Ewords_2Eword_mul\ A.27a)\ (ap\ (c_2Ewords_2Eword_2comp\ A.27a) \\ (ap\ (c_2Ewords_2En2w\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ V0w))) \end{aligned} \quad (48)$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0x \in (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V1y \in (ty_2EfcP_2Ecart \\
& 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp \\
& A_27a)\ V1y))\ V0x) = (ap\ (ap\ (c_2Ewords_2Eword_sub\ A_27a)\ V0x)\ V1y)))) \wedge \\
& ((\forall V2x \in (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V3y \in (ty_2EfcP_2Ecart \\
& 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ V2x)\ (ap\ (c_2Ewords_2Eword_2comp \\
& A_27a)\ V3y)) = (ap\ (ap\ (c_2Ewords_2Eword_sub\ A_27a)\ V2x)\ V3y)))) \wedge \\
& ((\forall V4x \in (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V5y \in (ty_2EfcP_2Ecart \\
& 2\ A_27a).(\forall V6z \in (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_add \\
& A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp \\
& A_27a)\ V4x))\ V5y))\ V6z) = (ap\ (ap\ (c_2Ewords_2Eword_sub\ A_27a) \\
& V6z)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ V4x)\ V5y)))))) \wedge ((\forall V7x \in \\
& (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V8y \in (ty_2EfcP_2Ecart\ 2 \\
& A_27a).(\forall V9z \in (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_add \\
& A_27a)\ V9z)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp \\
& A_27a)\ V7x))\ V8y)) = (ap\ (ap\ (c_2Ewords_2Eword_sub\ A_27a)\ V9z) \\
& (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ V7x)\ V8y)))))) \wedge ((\forall V10x \in \\
& (ty_2EfcP_2Ecart\ 2\ A_27b).((ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27b) \\
& (ap\ (c_2Ewords_2Eword_2comp\ A_27b)\ (ap\ (c_2Ewords_2En2w\ A_27b) \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \\
& V10x) = (ap\ (c_2Ewords_2Eword_2comp\ A_27b)\ V10x))) \wedge ((\forall V11x \in \\
& (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V12y \in (ty_2EfcP_2Ecart\ 2 \\
& A_27a).(\forall V13z \in (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_sub \\
& A_27a)\ V13z)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp \\
& A_27a)\ V11x))\ V12y)) = (ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ V13z) \\
& (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ V11x)\ V12y)))))) \wedge ((\forall V14x \in \\
& (ty_2EfcP_2Ecart\ 2\ A_27a).(\forall V15y \in (ty_2EfcP_2Ecart\ 2 \\
& A_27a).(\forall V16z \in (ty_2EfcP_2Ecart\ 2\ A_27a).((ap\ (ap\ (c_2Ewords_2Eword_sub \\
& A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mul\ A_27a)\ (ap\ (c_2Ewords_2Eword_2comp \\
& A_27a)\ V14x))\ V15y))\ V16z) = (ap\ (c_2Ewords_2Eword_2comp\ A_27a) \\
& (ap\ (ap\ (c_2Ewords_2Eword_add\ A_27a)\ (ap\ (ap\ (c_2Ewords_2Eword_mul \\
& A_27a)\ V14x)\ V15y))\ V16z)))))))))
\end{aligned}$$