

# thm\_2Ewords\_2EWORD\_\_SUB\_\_RZERO (TM- bZYymTgzYwf4gXk41wtUW496LmjZUQDJR)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Efc\_2E\_finite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efc\_2E\_finite\_image A0) \quad (1)$$

Let  $ty\_2Ebool\_2E\_itself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2E\_itself A0) \quad (2)$$

Let  $c\_2Ebool\_2E\_the\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2E\_the\_value A\_27a \in (ty\_2Ebool\_2E\_itself A\_27a) \quad (3)$$

Let  $ty\_2Eenum\_2E\_enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Eenum\_2E\_enum \quad (4)$$

Let  $c\_2Efc\_2E\_dimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Efc\_2E\_dimindex A\_27a \in (ty\_2Eenum\_2E\_enum^{(ty\_2Ebool\_2E\_itself A\_27a)}) \quad (5)$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_EF$



**Definition 17** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 18** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (13)$$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 20** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebo$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (14)$$

**Definition 21** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).(ap (ap c$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)})^{ty\_2Enum\_2Enum} \quad (15)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (16)$$

**Definition 22** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (17)$$

**Definition 23** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (18)$$

**Definition 24** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 26** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 27** We define  $c\_2EfcP\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

**Definition 28** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2EfcP\_2EFC$

**Definition 29** We define  $c\_2Ewords\_2Eword\_2comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).$

**Definition 30** We define  $c\_2Ewords\_2Eword\_2add$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V$

**Definition 31** We define  $c\_2Ewords\_2Eword\_2sub$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). ((ap\ (ap\ (c\_2Ewords\_2Eword\_2add\ A\_27a)\ V0w)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0)) = V0w)) \wedge (\forall V1w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). ((ap\ (ap\ (c\_2Ewords\_2Eword\_2add\ A\_27a)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0))\ V1w) = V1w))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0)) = (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0)) \tag{23}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). ((ap\ (ap\ (c\_2Ewords\_2Eword\_2sub\ A\_27a)\ V0w)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0)) = V0w))$$