

thm\_2Ewords\_2EWORD\_\_SUB\_\_TRIANGLE  
(TMWue8M2pvnZK8Mq6eX3CzN7EAqKFgMYDrH)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Efc\_2E\_finite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efc\_2E\_finite\_image A0) \quad (1)$$

Let  $ty\_2Ebool\_2E\_itself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2E\_itself A0) \quad (2)$$

Let  $c\_2Ebool\_2E\_the\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2E\_the\_value A\_27a \in (ty\_2Ebool\_2E\_itself A\_27a) \quad (3)$$

Let  $ty\_2Eenum\_2E\_enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Eenum\_2E\_enum \quad (4)$$

Let  $c\_2Efc\_2E\_dimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Efc\_2E\_dimindex A\_27a \in (ty\_2Eenum\_2E\_enum^{(ty\_2Ebool\_2E\_itself A\_27a)}) \quad (5)$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_EF$

**Definition 7** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (6)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then}$  (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebool\_2E\_3F\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap c\_Ebool\_2E\_2F\_5C$

**Definition 13** We define  $c\_2Efc\_2Efinite\_index$  to be  $\lambda A.27a : \iota.(ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum}$

Let  $ty\_2Efc\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efc\_2Ecart A0 A1) \quad (9)$$

Let  $c\_2Efc\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c\_2Efc\_2Edest\_cart A.27a A.27b \in ((A.27a^{(ty\_2Efc\_2Efinite\_image A.27b)})^{(ty\_2Efc\_2Ecart A.27a A.27b)}) \quad (10)$$

**Definition 14** We define  $c\_2Efc\_2Efc\_index$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in (ty\_2Efc\_2Ecart A.27a$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (11)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 16** We define  $c\_2Earithmetic\_2E\_2B$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 17** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 18** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 20** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebo$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 21** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).(ap (ap$

**Definition 22** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 23** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 24** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 26** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 27** We define  $c\_2EfcP\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

**Definition 28** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap (c\_2EfcP\_2EFC$

**Definition 29** We define  $c\_2Ewords\_2Eword\_add$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (18)$$

**Definition 30** We define  $c\_2Ewords\_2Eword\_2comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).$

**Definition 31** We define  $c\_2Ewords\_2Eword\_2sub$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(\forall V1w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(\forall V2x \in \\ (ty\_2Efc\_2Ecart\ 2\ A\_27a).((ap\ (ap\ (c\_2Ewords\_2Eword\_2sub\ A\_27a) \\ (ap\ (ap\ (c\_2Ewords\_2Eword\_2add\ A\_27a)\ V0v)\ V1w))\ V2x) = (ap\ (ap\ ( \\ c\_2Ewords\_2Eword\_2add\ A\_27a)\ V0v)\ (ap\ (ap\ (c\_2Ewords\_2Eword\_2sub \\ A\_27a)\ V1w)\ V2x))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(\forall V1w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(\forall V2x \in \\ (ty\_2Efc\_2Ecart\ 2\ A\_27a).((ap\ (ap\ (c\_2Ewords\_2Eword\_2sub\ A\_27a) \\ (ap\ (ap\ (c\_2Ewords\_2Eword\_2add\ A\_27a)\ V0v)\ V1w))\ V2x) = (ap\ (ap\ ( \\ c\_2Ewords\_2Eword\_2add\ A\_27a)\ (ap\ (ap\ (c\_2Ewords\_2Eword\_2sub \\ A\_27a)\ V0v)\ V2x))\ V1w)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(\forall V1v \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).((ap\ (ap\ (c\_2Ewords\_2Eword\_2sub \\ A\_27a)\ (ap\ (ap\ (c\_2Ewords\_2Eword\_2sub\ A\_27a)\ V1v)\ V0w))\ V1v) = ( \\ ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ V0w)))) \end{aligned} \quad (25)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V_0 v \in (\text{ty\_2EfcP\_2Ecart} \\ & \quad 2 \ A_{.27a}). (\forall V_1 w \in (\text{ty\_2EfcP\_2Ecart } 2 \ A_{.27a}). (\forall V_2 x \in \\ & \quad (\text{ty\_2EfcP\_2Ecart } 2 \ A_{.27a}). ((\text{ap } (\text{ap } (\text{c\_2Ewords\_2Eword\_add } A_{.27a}) \\ & \quad (\text{ap } (\text{ap } (\text{c\_2Ewords\_2Eword\_sub } A_{.27a}) \ V_0 v) \ V_1 w)) \ (\text{ap } (\text{ap } (\text{c\_2Ewords\_2Eword\_sub} \\ & \quad A_{.27a}) \ V_1 w) \ V_2 x)) = (\text{ap } (\text{ap } (\text{c\_2Ewords\_2Eword\_sub } A_{.27a}) \ V_0 v) \ V_2 x)))))) \end{aligned}$$