

thm\_2Ewords\_2Ebit\_\_count\_\_is\_\_zero  
(TMK45A9eE9nmqmtA7eUM6S1NWt85yqPjxp7)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$ .

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (1)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (2)$$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty ty\_2Eenum\_2Eenum \quad (3)$$

Let  $c\_2Efcf\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Efcf\_2Edimindex A\_27a \in (ty\_2Eenum\_2Eenum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (4)$$

Let  $c\_2Eenum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EZERO\_REP \in \omega \quad (5)$$

Let  $c\_2Eenum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EABS\_num \in (ty\_2Eenum\_2Eenum^{\omega}) \quad (6)$$

**Definition 3** We define  $c\_2Eenum\_2E0$  to be  $(ap c\_2Eenum\_2EABS\_num c\_2Eenum\_2EZERO\_REP)$ .

**Definition 4** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Eenum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num ($

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic\_2E\_2B$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $ty\_2Efc\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Efc\_2Efinite\_image A0) \quad (10)$$

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2. V0t))$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21$

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. V2t$

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 15** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. V1n$

**Definition 16** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap c\_2Ebool\_2E\_2F\_5C$

**Definition 17** We define  $c\_2Efc\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efc\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty\_2Efc\_2Ecart A0 A1) \quad (11)$$

Let  $c\_2Efc\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Efc\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{(ty\_2Efc\_2Efinite\_image A\_27b)})^{(ty\_2Efc\_2Ecart A\_27a A\_27b)}) \quad (12)$$

**Definition 18** We define  $c\_2Efc\_2Efc\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efc\_2Ecart\ A\_27a)$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 20** We define  $c\_2Ewords\_2Ebit\_count\_upto$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1w$

**Definition 21** We define  $c\_2Ewords\_2Ebit\_count$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(ap$

**Definition 22** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (14)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 23** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 24** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 26** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 27** We define  $c\_2Efc\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum})).(ap$

**Definition 28** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Efc\_2EFC$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). ((V0w = (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0)) \Leftrightarrow ( \\ & \forall V1i \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1i) \\ & (ap\ (c\_2Efc\_2Edimindex\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a)))) \Rightarrow \\ & (\neg(p\ (ap\ (ap\ (c\_2Efc\_2Efc\_index\ 2\ A\_27a)\ V0w)\ V1i))))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0n \in ty\_2Enum\_2Enum. ( \\ & \forall V1w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). (((ap\ (ap\ (c\_2Ewords\_2Ebit\_count\_upto \\ & A\_27a)\ V0n)\ V1w) = c\_2Enum\_2E0) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum. \\ & ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V2i)\ V0n)) \Rightarrow (\neg(p\ (ap\ (ap\ (c\_2Efc\_2Efc\_index \\ & 2\ A\_27a)\ V1w)\ V2i)))))))) \end{aligned} \quad (22)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a). (((ap\ (c\_2Ewords\_2Ebit\_count\ A\_27a)\ V0w) = c\_2Enum\_2E0) \Leftrightarrow \\ & (V0w = (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0)))) \end{aligned}$$