

thm_2Ewords_2Ebit__count__upto__SUC (TMFwZjTDZrDyvrCrmnBPx9ikQxxKaT4hw4e)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{5}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2)) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Definition 7 We define c_Enum_2E0 to be $(ap\ c_Enum_2EABS_num\ c_Enum_2EZERO_REP)$.

Definition 8 We define $c_Earithmic_2EZERO$ to be c_Enum_2E0 .

Definition 9 We define $c_Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_Earithmic_2EBIT1\ n))$.

Definition 10 We define $c_Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Efc_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efc_2Efinite_image\ A0) \quad (7)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (8)$$

Let $c_Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (9)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (10)$$

Definition 11 We define c_Ebool_2EF to be $(ap\ (c_Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 We define c_Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_2E3D_3D_3E\ V0t)\ c_Ebool_2E7E))$.

Definition 13 We define c_Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A)\ p)$ of type $\iota \Rightarrow \iota$.

Definition 14 We define c_Ebool_2E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_Emin_2E40\ A_27a)\ P)))$.

Definition 15 We define $c_Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0n$.

Definition 16 We define $c_Ebool_2E3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_Ebool_2E2F_5C\ A_27a)\ P)))$.

Definition 17 We define $c_2Efc_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_Emin_2E40\ (A_27a^{ty_2Enum_2Enum})))$.

Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efc_2Ecart\ A0\ A1) \quad (11)$$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})^{(ty_2Efc_2Ecart\ A_27a\ A_27b)}) \quad (12)$$

Definition 18 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A_27a)$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 20 We define $c_2Ewords_2Ebit_count_upto$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.\lambda V1w$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap (ap c_2Earithmetic_2E_2B\ V1n)\ V0m)))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B\ V0m)\ V2p) = (ap (ap c_2Earithmetic_2E_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n)))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (17)$$

Assume the following.

$$((\forall V0f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). ((ap (ap c_2Esum_num_2ESUM\ c_2Enum_2E0)\ V0f) = c_2Enum_2E0)) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). ((ap (ap c_2Esum_num_2ESUM\ (ap\ c_2Enum_2ESUC\ V1m))\ V2f) = (ap (ap c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Esum_num_2ESUM\ V1m)\ V2f))\ (ap\ V2f\ V1m))))))) \quad (18)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Efc_2Ecart\ 2\ A_27a). (\forall V1n \in ty_2Enum_2Enum. ((ap (ap (c_2Ewords_2Ebit_count_upto\ A_27a)\ (ap\ c_2Enum_2ESUC\ V1n))\ V0w) = (ap (ap c_2Earithmetic_2E_2B\ (ap (ap (ap (c_2Ebool_2ECOND\ ty_2Enum_2Enum)\ (ap (ap (c_2Efc_2Efc_index\ 2\ A_27a)\ V0w)\ V1n))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))\ c_2Enum_2E0))\ (ap (ap (c_2Ewords_2Ebit_count_upto\ A_27a)\ V1n)\ V0w))))))$$