

thm_2Ewords_2Edimindex_4
(TMUXX1J1HLAWLM1LorHU3thx6DycdyDV5uh)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A.\lambda a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda V2t2 \in A.\lambda V3t3 \in A.$

Let $ty_2EfcP_2Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit0\ A0) \tag{3}$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \tag{4}$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (5)$$

Let $c_2Efcf_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcf_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (6)$$

Definition 10 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2E2ET)$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (7)$$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ V2t\ t1))))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (9)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Ebool_2E_21\ 2)\ (ap\ V1y\ V0x)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (10)$$

Definition 14 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (ap\ V1s\ V0x)))$

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2E2EF)$.

Definition 16 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ (ap\ V0s\ V0s)))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (13)$$

Definition 17 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 18 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (15)$$

Definition 19 We define c_2Enum_2ESUC to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 20 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 21 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 22 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 23 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ (\\ & ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))) \\ & V0n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V0n))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\ & A_27a.(((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((ap\ (c.2EfcP.2Edimindex\ (ty.2EfcP.2Ebit0 \\ & A.27a))\ (c.2Ebool.2Ethe_value\ (ty.2EfcP.2Ebit0\ A.27a))) = (\\ & ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ ty.2Enum.2Enum)\ (ap\ (c.2Epred_set.2EFINITE \\ & A.27a)\ (c.2Epred_set.2EUNIV\ A.27a)))\ (ap\ (ap\ c.2Earithmetic.2E_2A \\ & (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) \\ & (ap\ (c.2EfcP.2Edimindex\ A.27a)\ (c.2Ebool.2Ethe_value\ A.27a)))) \\ & (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((p\ (ap\ (c.2Epred_set.2EFINITE \\ & (ty.2EfcP.2Ebit0\ A.27a))\ (c.2Epred_set.2EUNIV\ (ty.2EfcP.2Ebit0 \\ & A.27a)))) \Leftrightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (c.2Epred_set.2EUNIV \\ & A.27a)))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} ((ap\ (c.2EfcP.2Edimindex\ ty.2Eone.2Eone)\ (c.2Ebool.2Ethe_value \\ ty.2Eone.2Eone)) = (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\ c.2Earithmetic.2EZERO))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} (p\ (ap\ (c.2Epred_set.2EFINITE\ ty.2Eone.2Eone)\ (c.2Epred_set.2EUNIV \\ ty.2Eone.2Eone))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty.2Enum.2Enum.(((ap\ c.2Enumeral.2EiDUB\ (ap\ c.2Earithmetic.2EBIT1 \\ V0n)) = (ap\ c.2Earithmetic.2EBIT2\ (ap\ c.2Enumeral.2EiDUB\ V0n))) \wedge \\ (((ap\ c.2Enumeral.2EiDUB\ (ap\ c.2Earithmetic.2EBIT2\ V0n)) = (ap \\ c.2Earithmetic.2EBIT2\ (ap\ c.2Earithmetic.2EBIT1\ V0n))) \wedge ((ap \\ c.2Enumeral.2EiDUB\ c.2Earithmetic.2EZERO) = c.2Earithmetic.2EZERO)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} (\forall V0i \in ty.2Enum.2Enum.((ap\ c.2Enumeral.2EiDUB\ (ap\ c.2Earithmetic.2ENUMERAL \\ V0i)) = (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Enumeral.2EiDUB\ V0i)))) \end{aligned} \quad (25)$$

Theorem 1

$$\begin{aligned} ((ap\ (c.2EfcP.2Edimindex\ (ty.2EfcP.2Ebit0\ (ty.2EfcP.2Ebit0 \\ ty.2Eone.2Eone)))\ (c.2Ebool.2Ethe_value\ (ty.2EfcP.2Ebit0 \\ (ty.2EfcP.2Ebit0\ ty.2Eone.2Eone)))) = (ap\ c.2Earithmetic.2ENUMERAL \\ (ap\ c.2Earithmetic.2EBIT2\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))) \end{aligned}$$