

thm_2Ewords_2Edimword__11
(TMH6ycDTtY5TgF2HGVYm39frHfkmhjs3cVu)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (ap c_2Enum_2EREP_num m))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_Emin_2E_40$

Definition 11 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (5)$$

Let $c_Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 16 We define $c_Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 17 We define c_Enum_2E0 to be $(ap\ c_Enum_2EABS_num\ c_Enum_2EZERO_REP)$.

Let $c_Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (8)$$

Let $c_Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_Enumeral_2Eonecount : \iota$ be given. Assume the following.

$$c_Enumeral_2Eonecount \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (10)$$

Let $c_Enumeral_2Exactlog : \iota$ be given. Assume the following.

$$c_Enumeral_2Exactlog \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (11)$$

Definition 18 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 19 We define $c_Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_Ebool_2E$

Definition 20 We define $c_Earithmetic_2EZERO$ to be c_Enum_2E0 .

Definition 21 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_Enum_Enum.(ap (ap c_Earithmetic$

Definition 22 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (12)$$

Definition 23 We define $c_Earithmetic_EDIV2$ to be $\lambda V0n \in ty_Enum_Enum.(ap (ap c_Earithmetic$

Let $c_Enumeral_Eteexp_help : \iota$ be given. Assume the following.

$$c_Enumeral_Eteexp_help \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (13)$$

Let $c_Earithmetic_EODD : \iota$ be given. Assume the following.

$$c_Earithmetic_EODD \in (2^{ty_Enum_Enum}) \quad (14)$$

Definition 24 We define c_Ebool_ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27$

Let $c_Earithmetic_E_2A : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2A \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (15)$$

Definition 25 We define $c_Enumeral_Einternal_mult$ to be $c_Earithmetic_E_2A$.

Let $ty_Ebool_Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_Ebool_Eitself A0) \quad (16)$$

Let $c_Ewords_Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ewords_Edimword A_27a \in (ty_Enum_Enum^{(ty_Ebool_Eitself A_27a)}) \quad (17)$$

Definition 26 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap (ap c_Earithmetic$

Let $c_Ebool_Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ebool_Ethe_value A_27a \in (ty_Ebool_Eitself A_27a) \quad (18)$$

Let $ty_Eone_Eone : \iota$ be given. Assume the following.

$$nonempty ty_Eone_Eone \quad (19)$$

Let $ty_EfcP_Ebit0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_EfcP_Ebit0 A0) \quad (20)$$

Let $ty_2EfcP_2Ebit1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Ebit1\ A0) \quad (21)$$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MIN\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))))) \quad (27)$$

Assume the following.

$$(((ap\ c_2Enum_2ESUC\ c_2Earithmetic_2EZERO) = (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ V0n)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Enum_2ESUC\ (ap\ c_2Earithmetic_2EBIT2\ V1n)) = (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enum_2ESUC\ V1n))))))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c_2Eprim_rec_2EPRE\ c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge \\
& (((ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmic_2EBIT1\ c_2Earithmic_2EZERO)) = \\
& c_2Earithmic_2EZERO) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\
& c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmic_2EBIT1\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) = (ap\ c_2Earithmic_2EBIT2\ (ap\ c_2Eprim_rec_2EPRE\ (ap \\
& c_2Earithmic_2EBIT1\ V0n)))))) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& ((ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmic_2EBIT1\ (ap\ c_2Earithmic_2EBIT2 \\
& V1n))) = (ap\ c_2Earithmic_2EBIT2\ (ap\ c_2Earithmic_2EBIT1 \\
& V1n)))) \wedge ((\forall V2n \in ty_2Enum_2Enum. ((ap\ c_2Eprim_rec_2EPRE \\
& (ap\ c_2Earithmic_2EBIT2\ V2n)) = (ap\ c_2Earithmic_2EBIT1\ V2n))))))))) \\
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p\ (ap\ c_2Earithmic_2EVEN\ c_2Earithmic_2EZERO)) \wedge \\
& ((p\ (ap\ c_2Earithmic_2EVEN\ (ap\ c_2Earithmic_2EBIT2\ V0n))) \wedge \\
& ((\neg(p\ (ap\ c_2Earithmic_2EVEN\ (ap\ c_2Earithmic_2EBIT1\ V0n)))) \wedge \\
& ((\neg(p\ (ap\ c_2Earithmic_2EODD\ c_2Earithmic_2EZERO)) \wedge ((\\
& \neg(p\ (ap\ c_2Earithmic_2EODD\ (ap\ c_2Earithmic_2EBIT2\ V0n)))) \wedge \\
& (p\ (ap\ c_2Earithmic_2EODD\ (ap\ c_2Earithmic_2EBIT1\ V0n))))))))) \\
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0acc \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap\ (ap\ c_2Enumeral_2Eexp_help\ c_2Earithmic_2EZERO\ V0acc) = \\
& (ap\ c_2Earithmic_2EBIT2\ V0acc)) \wedge (((ap\ (ap\ c_2Enumeral_2Eexp_help \\
& (ap\ c_2Earithmic_2EBIT1\ V1n))\ V0acc) = (ap\ (ap\ c_2Enumeral_2Eexp_help \\
& (ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Earithmic_2EBIT1\ V1n)))\ (ap \\
& c_2Earithmic_2EBIT1\ V0acc))) \wedge ((ap\ (ap\ c_2Enumeral_2Eexp_help \\
& (ap\ c_2Earithmic_2EBIT2\ V1n))\ V0acc) = (ap\ (ap\ c_2Enumeral_2Eexp_help \\
& (ap\ c_2Earithmic_2EBIT1\ V1n))\ (ap\ c_2Earithmic_2EBIT1\ V0acc)))))) \\
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
& c_2Earithmetic_2EZERO) V0x) = V0x)) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
& (ap c_2Earithmetic_2EBIT1 V1n)) V2x) = (ap (ap c_2Enumeral_2Eonecount \\
& V1n) (ap c_2Enum_2ESUC V2x)))))) \wedge ((\forall V3n \in ty_2Enum_2Enum. \\
& (\forall V4x \in ty_2Enum_2Enum.((ap (ap c_2Enumeral_2Eonecount \\
& (ap c_2Earithmetic_2EBIT2 V3n)) V4x) = c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Enumeral_2Exactlog c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge \\
& ((\forall V0n \in ty_2Enum_2Enum.((ap c_2Enumeral_2Exactlog (\\
& ap c_2Earithmetic_2EBIT1 V0n)) = c_2Earithmetic_2EZERO)) \wedge ((\forall V1n \in \\
& ty_2Enum_2Enum.((ap c_2Enumeral_2Exactlog (ap c_2Earithmetic_2EBIT2 \\
& V1n)) = (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) \\
& (\lambda V2x \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap (ap (c_2Emin_2E_3D ty_2Enum_2Enum) V2x) c_2Earithmetic_2EZERO)) \\
& c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 V2x)))) (ap \\
& (ap c_2Enumeral_2Eonecount V1n) c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{35}$$

Assume the following.

$$(\forall V0x \in ty_2Enum_2Enum.((ap c_2Earithmetic_2EDIV2 (ap c_2Earithmetic_2EBIT1 V0x)) = V0x)) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Enum_2Enum. (\\
& \forall V2y \in ty_2Enum_2Enum. (((ap (ap c_2Earithmic_2E_2A c_2Earithmic_2EZERO) \\
& V0n) = c_2Earithmic_2EZERO) \wedge (((ap (ap c_2Earithmic_2E_2A \\
& V0n) c_2Earithmic_2EZERO) = c_2Earithmic_2EZERO) \wedge ((ap \\
& (ap c_2Earithmic_2E_2A (ap c_2Earithmic_2EBIT1 V1x)) (ap \\
& c_2Earithmic_2EBIT1 V2y)) = (ap (ap c_2Enumeral_2Einternal_mult \\
& (ap c_2Earithmic_2EBIT1 V1x)) (ap c_2Earithmic_2EBIT1 V2y)))) \wedge \\
& (((ap (ap c_2Earithmic_2E_2A (ap c_2Earithmic_2EBIT1 V1x)) \\
& (ap c_2Earithmic_2EBIT2 V2y)) = (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum \\
& ty_2Enum_2Enum) (\lambda V3n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND \\
& ty_2Enum_2Enum) (ap c_2Earithmic_2EODD V3n)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V3n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 \\
& V1x)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT1 \\
& V1x)) (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V2y)))) \wedge (((ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2EBIT2 V1x)) (ap c_2Earithmic_2EBIT1 V2y)) = \\
& (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V4m \in \\
& ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap c_2Earithmic_2EODD V4m)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V4m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT1 \\
& V2y)))) (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT2 \\
& V1x)) (ap c_2Earithmic_2EBIT1 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V1x)))) \wedge ((ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2EBIT2 V1x)) (ap c_2Earithmic_2EBIT2 V2y)) = \\
& (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V5m \in \\
& ty_2Enum_2Enum. (ap (ap (c_2Ebool_2ELET ty_2Enum_2Enum ty_2Enum_2Enum) \\
& (\lambda V6n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) \\
& (ap c_2Earithmic_2EODD V5m)) (ap (ap c_2Enumeral_2Eexp_help \\
& (ap c_2Earithmic_2EDIV2 V5m)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT2 \\
& V2y)))) (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap c_2Earithmic_2EODD \\
& V6n)) (ap (ap c_2Enumeral_2Eexp_help (ap c_2Earithmic_2EDIV2 \\
& V6n)) (ap c_2Eprim_rec_2EPRE (ap c_2Earithmic_2EBIT2 V1x)))) \\
& (ap (ap c_2Enumeral_2Einternal_mult (ap c_2Earithmic_2EBIT2 \\
& V1x)) (ap c_2Earithmic_2EBIT2 V2y)))))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V2y)))) (ap c_2Enumeral_2Eexactlog \\
& (ap c_2Earithmic_2EBIT2 V1x))))))))) (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow ((ap (c_2Ewords_2Edimword A_27a) \\
& (c_2Ebool_2Ethe_value A_27a)) = (ap (ap c_2Earithmic_2E_2A \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 c_2Earithmic_2EZERO))) \\
& (ap (c_2Ewords_2EINT_MIN A_27a) (c_2Ebool_2Ethe_value A_27a)))) (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((ap (c_2Ewords_2EINT_MIN (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 \\
& \quad (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\
& \quad (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))))))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& ((ap (c_2Ewords_2Edimword (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 \\
& \quad (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))) (c_2Ebool_2Ethe_value \\
& \quad (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit1 (ty_2Efc_2Ebit0 ty_2Eone_2Eone)))))) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\
& \quad (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))))))))))))
\end{aligned}$$