

# thm\_2Ewords\_2Efinite\_\_10 (TMYwhiqePVGGE- FjWTPVThCmaSGQmiXdBN8oc)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2EfcP\_2Ebit0 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2EfcP\_2Ebit0 A0) \quad (1)$$

Let  $ty\_2EfcP\_2Ebit1 : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2EfcP\_2Ebit1 A0) \quad (2)$$

**Definition 8** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_27E)$ .

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \quad (3)$$

**Definition 9** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 10** We define  $c\_2Ebool\_2E\_25C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (4)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (5)$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (6)$$

**Definition 12** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EEMPTY\ A\_27a\ V1s))$

**Definition 13** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

**Definition 14** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap\ (c\_2Ebool\_2E\_21\ A\_27a\ V0s))$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ (ty\_2Efc\_2Ebit0\ A\_27a))\ (c\_2Epred\_set\_2EUNIV\ (ty\_2Efc\_2Ebit0\ A\_27a)))) \Leftrightarrow (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ (c\_2Epred\_set\_2EUNIV\ A\_27a)))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ (ty\_2Efc\_2Ebit1\ A\_27a))\ (c\_2Epred\_set\_2EUNIV\ (ty\_2Efc\_2Ebit1\ A\_27a)))) \Leftrightarrow (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ (c\_2Epred\_set\_2EUNIV\ A\_27a)))) \quad (10)$$

Assume the following.

$$(p\ (ap\ (c\_2Epred\_set\_2EFINITE\ ty\_2Eone\_2Eone)\ (c\_2Epred\_set\_2EUNIV\ ty\_2Eone\_2Eone))) \quad (11)$$

**Theorem 1**

$(p (ap (c\_2Epred\_set\_2EFINITE (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone)))) (c\_2Epred\_set\_2EUNIV (ty\_2Efc\_2Ebit0 (ty\_2Efc\_2Ebit1 (ty\_2Efc\_2Ebit0 ty\_2Eone\_2Eone))))))$