

thm\_2Ewords\_2Efoldl\_reduce\_xnor  
 (TMaRT8RCiwVve61uXSracHYmeRiQ8aLEEUz)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 4** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A\_27b) A\_27c) A\_27b)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (2)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (3)$$

Let  $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Elist\_2EMAP A\_27a A\_27b \in (((ty\_2Elist\_2Elist A\_27b)^{(ty\_2Elist\_2Elist A\_27a)})^{(A\_27b^{A\_27a})}) \quad (4)$$

Let  $c\_2Elist\_2ENULL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENULL A\_27a \in (2^{(ty\_2Elist\_2Elist A\_27a)}) \quad (5)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (6)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 5** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) (\lambda V1t \in 2.V1t)) P))$

**Definition 7** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ap (c\_2Ebool\_2E\_7E V2t) c\_2Ebool\_2EF))))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (11)$$

**Definition 11** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num m)$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p x)) \text{ else } (\lambda x.x \in A \wedge \neg p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40)))$

**Definition 14** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Eprim\_rec\_2E\_3C m n))$

**Definition 15** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Earithmetic\_2E\_3E m n))$

**Definition 16** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ap (c\_2Ebool\_2E\_7E V2t) c\_2Ebool\_2EF))))))$

**Definition 17** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Earithmetic\_2E\_3E\_3D m n))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (12)$$

**Definition 18** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a.))$

**Definition 20** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (13)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (14)$$

**Definition 21** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2ESUC\ (ap\ (c\_2Enum\_2EiiSUC\ A\_27n)))$

**Definition 22** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (15)$$

**Definition 23** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ (c\_2Earithmetic\_2EBIT2\ A\_27n)))$

Let  $ty\_2Efcp\_2Efinit\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinit\_image\ A0) \quad (16)$$

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (17)$$

Let  $c\_2Ebool\_2Ethel\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethel\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a)^{ty\_2Ebool\_2Eitself\ A\_27a} \quad (18)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)})^{ty\_2Enum\_2Enum} \quad (19)$$

**Definition 24** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Ebool\_2E\_2F\_5C\ A\_27a)))$

**Definition 25** We define  $c\_2Efcp\_2Efinit\_index$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Emin\_2E\_40\ (A\_27a^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_0.\text{nonempty } A_0 \Rightarrow & \forall A_1.\text{nonempty } A_1 \Rightarrow \text{nonempty } (ty\_2Efcp\_2Ecart \\ & A_0 A_1) \end{aligned} \quad (20)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart \\ & A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efcp\_index\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \end{aligned} \quad (21)$$

**Definition 26** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)$

**Definition 27** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap\ g\ A\_27b))$

**Definition 28** We define  $c\_2Ewords\_2Eword\_xnor$  to be  $\lambda A\_27a : \iota. \lambda V0v \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\lambda V1x \in$

Let  $c\_2Elist\_2EGENLIST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & c\_2Elist\_2EGENLIST\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Enum\_2Enum)})^{(A\_27a^{ty\_2Enum\_2Enum})}) \end{aligned} \quad (22)$$

Let  $c\_2Elist\_2ETL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & c\_2Elist\_2ETL\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \end{aligned} \quad (23)$$

Let  $c\_2Elist\_2EHD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2EHD\ A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (24)$$

Let  $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Elist\_2EFOLDL \\ & A\_27a A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \end{aligned} \quad (25)$$

**Definition 29** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27a\ .(V1x \in A\_27b)))$

**Definition 30** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 31** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2EFOLDL\ n\ 1)\ 0)$

**Definition 32** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (26)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Eone\_2Eone \quad (27)$$

**Definition 33** We define  $c\_2Ewords\_2Eword\_reduce$  to be  $\lambda A\_27a : \iota. \lambda V0f \in ((2^2)^2). \lambda V1w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). ap (ap (c\_2Ewords\_2Eword\_reduce\ A\_27a) f) w$

**Definition 34** We define  $c\_2Ewords\_2Ereduce\_xnor$  to be  $\lambda A\_27a : \iota. (ap (c\_2Ewords\_2Eword\_reduce\ A\_27a) \_xnor) a$

**Definition 35** We define  $c\_2Earithmetic\_2EMIN$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. ap (c\_2Ewords\_2Eword\_reduce\ A\_27a) m$

**Definition 36** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. ap (c\_2Ewords\_2Eword\_reduce\ A\_27a) m$

**Definition 37** We define  $c\_2Ewords\_2Eword\_bits$  to be  $\lambda A\_27a : \iota. \lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. ap (c\_2Ewords\_2Eword\_reduce\ A\_27a) h$

**Definition 38** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1g \in (A\_27c^{A\_27b}). ap (ap (c\_2Ewords\_2Eword\_reduce\ A\_27a) f) g$

**Definition 39** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. ap (ap (ap (c\_2Ebit\_2ESBIT\ b) n) V0b) V1n$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}})})^{ty\_2Enum\_2Enum}) \quad (28)$$

**Definition 40** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). ap (ap (c\_2Ewords\_2Eword\_reduce\ A\_27a) w)$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (29)$$

**Definition 41** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. ap (c\_2Ebit\_2EDIV\ x) n$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (30)$$

**Definition 42** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. ap (c\_2Ebit\_2EMOD\ x) n$

**Definition 43** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum. \lambda V1l \in ty\_2Enum\_2Enum. ap (c\_2Ebit\_2EBIT\ h) l$

**Definition 44** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. ap (c\_2Ebit\_2EBITS\ b) n$

**Definition 45** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. ap (ap (c\_2Ewords\_2Eword\_reduce\ A\_27a) n) V0n$

**Definition 46** We define  $c\_2Ewords\_2Ew2w$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a). ap (c\_2Ewords\_2Eword\_reduce\ A\_27b) w$

**Definition 47** We define  $c\_2Ewords\_2Eword\_extract$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0h \in ty\_2Enum\_2Enum. ap (c\_2Ewords\_2Eword\_extract\ A\_27a) h$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap (c\_2Earithmetic\_2E\_2B\ V0m) c\_2Enum\_2E0) = V0m))) \quad (31)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\
 & ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
 & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
 & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))
 \end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V1n) V0m)))
 \end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p))))
 \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap c\_2Enum\_2ESUC V0m)) V1n))))
 \end{aligned} \tag{35}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 c\_2Enum\_2E0) V0n))) \tag{36}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & V1n) V0m))))
 \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2D \\
 c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge ((ap (ap c\_2Earithmetic\_2E\_2D \\
 V0m) c\_2Enum\_2E0) = V0m)))
 \end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n)))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{40}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \tag{41} \\
V0m) V0m)))$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Enum\_2ESUC V1n)) V0m))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & V1n)) V0m))))))) \\
 \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
 & c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & c\_2Earithmetic\_2EZERO)) V0n))) \\
 \end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2D (ap \\
 & (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p) = (ap (ap c\_2Earithmetic\_2E\_2D \\
 & V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))))))) \\
 \end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
 & \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap (ap c\_2Earithmetic\_2E\_2D V0m) V1n)) V2p)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & V0m) (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p))))))) \\
 \end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (\forall V1a \in ty\_2Enum\_2Enum. \\
 & (\forall V2b \in ty\_2Enum\_2Enum. ((p (ap V0P (ap (ap c\_2Earithmetic\_2E\_2D \\
 & V1a) V2b)) \Leftrightarrow (\forall V3d \in ty\_2Enum\_2Enum. (((V2b = (ap (ap c\_2Earithmetic\_2E\_2D \\
 & V1a) V3d)) \Rightarrow (p (ap V0P c\_2Enum\_2E0))) \wedge ((V1a = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \\
 \end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
 & \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & (ap (ap c\_2Earithmetic\_2EMIN V1m) V0n)) V2p)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & V1m) V2p)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V2p)))) \wedge (( \\
 & p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2p) (ap (ap c\_2Earithmetic\_2EMIN \\
 & V1m) V0n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2p) V1m)) \wedge (p \\
 & (ap (ap c\_2Earithmetic\_2E\_3C\_3D V2p) V0n))))))) \\
 \end{aligned} \tag{50}$$

Assume the following.

$$True \tag{51}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & \forall V0f \in (A\_27b^{A\_27a}). (\forall V1x \in A\_27a. ((ap (ap (c\_2Ebool\_2ELET \\ & A\_27a A\_27b) V0f) V1x) = (ap V0f V1x))) \end{aligned} \quad (54)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (55)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\ & ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (56)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (57)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (61)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (62)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (63)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (64)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (70)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False)) \quad (71)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (72)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1) \wedge (\neg(p V1t2))))))) \quad (73)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\ & \text{nonempty } A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27a^{A\_27c}). \\ & (\forall V2x \in A\_27c. ((ap (ap (ap (c\_2Ecombin\_2Eo A\_27c A\_27b A\_27a) \\ & V0f) V1g) V2x) = (ap V0f (ap V1g V2x))))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap (ap (c\_2Ecombin\_2EK \\ & A\_27a A\_27b) V0x) V1y) = V0x))) \end{aligned} \quad (76)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2EI \\ & A\_27a) V0x) = V0x)) \quad (77)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) \\ & (ap (c\_2Efcp\_2Edimindex A\_27a) (c\_2Ebool\_2Ethe\_value A\_27a)))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b). (\forall V1y \in (ty\_2Efcp\_2Ecart \\ & A\_27a A\_27b). ((V0x = V1y) \Leftrightarrow (\forall V2i \in ty\_2Enum\_2Enum. ((p (ap \\ & (ap c\_2Eprim\_rec\_2E\_3C V2i) (ap (c\_2Efcp\_2Edimindex A\_27b) ( \\ & c\_2Ebool\_2Ethe\_value A\_27b)))) \Rightarrow ((ap (ap (c\_2Efcp\_2Efcp\_index \\ & A\_27a A\_27b) V0x) V2i) = (ap (ap (c\_2Efcp\_2Efcp\_index A\_27a A\_27b) \\ & V1y) V2i))))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (\forall V1i \in ty\_2Enum\_2Enum. \\ & ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2Efcp\_2Edimindex A\_27b) \\ & (c\_2Ebool\_2Ethe\_value A\_27b)))) \Rightarrow ((ap (ap (c\_2Efcp\_2Efcp\_index \\ & A\_27a A\_27b) (ap (c\_2Efcp\_2EFCP A\_27a A\_27b) V0g)) V1i) = (ap V0g \\ & V1i)))))) \end{aligned} \quad (80)$$

Assume the following.

$$((ap (c_2Efcop_2Edimindex ty_2Eone_2Eone) (c_2Ebool_2Ethethe_value ty_2Eone_2Eone)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \quad (81)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow ( \\ & (\forall V0f \in (A_27b^{A_27a}). ((ap (ap (c_2Elist_2EMAP A_27a A_27b) \\ & V0f) (c_2Elist_2ENIL A_27a)) = (c_2Elist_2ENIL A_27b))) \wedge (\forall V1f \in \\ & (A_27b^{A_27a}). (\forall V2h \in A_27a. (\forall V3t \in (ty_2Elist_2Elist \\ & A_27a). ((ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) (ap (ap (c_2Elist_2ECONS \\ & A_27a) V2h) V3t)) = (ap (ap (c_2Elist_2ECONS A_27b) (ap V1f V2h)) \\ & (ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) V3t))))))) \end{aligned} \quad (82)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow ( \\ & (\forall V0f \in ((A_27b^{A_27a})^{A_27b}). (\forall V1e \in A_27b. ((ap ( \\ & ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V0f) V1e) (c_2Elist_2ENIL \\ & A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27a})^{A_27b}). (\forall V3e \in \\ & A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist A_27a). \\ & ((ap (ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V2f) V3e) (ap (ap (c_2Elist_2ECONS \\ & A_27a) V4x) V5l)) = (ap (ap (ap (c_2Elist_2EFOLDL A_27a A_27b) V2f) \\ & (ap (ap V2f V3e) V4x)) V5l))))))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist A_27a)}). \\ & (((p (ap V0P (c_2Elist_2ENIL A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a). ((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_27a. (p (ap V0P (ap (ap ( \\ & c_2Elist_2ECONS A_27a) V2h) V1t))))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a). (p (ap V0P V3l)))))) \end{aligned} \quad (84)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow ( \\ & \forall V0l \in (ty_2Elist_2Elist A_27a). (\forall V1f \in (A_27b^{A_27a}). \\ & ((\neg(p (ap (c_2Elist_2ENULL A_27a) V0l))) \Rightarrow ((ap (ap (c_2Elist_2EMAP \\ & A_27a A_27b) V1f) (ap (c_2Elist_2ETL A_27a) V0l)) = (ap (c_2Elist_2ETL \\ & A_27b) (ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) V0l))))))) \end{aligned} \quad (85)$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow \\
 & \quad \forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1g \in (A_{27a}^{ty\_2Enum\_2Enum}). \\
 & \quad (\forall V2n \in ty\_2Enum\_2Enum.((ap (ap (c_2Elist\_2EMAP\ A_{27a}\ A_{27b}) \\
 & \quad V0f) (ap (ap (c_2Elist\_2EGENLIST\ A_{27a})\ V1g)\ V2n)) = (ap (ap (c_2Elist\_2EGENLIST\ \\
 & \quad A_{27b}) (ap (ap (c_2Ecombin\_2Eo\ ty\_2Enum\_2Enum\ A_{27b}\ A_{27a})\ V0f) \\
 & \quad V1g))\ V2n)))))) \\
 & \tag{86}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. \\
 & \quad \forall V1f \in (A_{27a}^{ty\_2Enum\_2Enum}).((p (ap (ap c_2Eprim\_rec\_2E\_3C \\
 & \quad c\_2Enum\_2E0)\ V0n)) \Rightarrow ((ap (c_2Elist\_2EHD\ A_{27a}) (ap (ap (c_2Elist\_2EGENLIST\ \\
 & \quad A_{27a})\ V1f)\ V0n)) = (ap V1f\ c\_2Enum\_2E0)))))) \\
 & \tag{87}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. \\
 & \quad \forall V1f \in (A_{27a}^{ty\_2Enum\_2Enum}).(\forall V2g \in (A_{27a}^{ty\_2Enum\_2Enum}). \\
 & \quad (((ap (ap (c_2Elist\_2EGENLIST\ A_{27a})\ V1f)\ V0n) = (ap (ap (c_2Elist\_2EGENLIST\ \\
 & \quad A_{27a})\ V2g)\ V0n)) \Leftrightarrow (\forall V3x \in ty\_2Enum\_2Enum.((p (ap (ap c_2Eprim\_rec\_2E\_3C \\
 & \quad V3x)\ V0n)) \Rightarrow ((ap V1f\ V3x) = (ap V2g\ V3x))))))) \\
 & \tag{88}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. \\
 & \quad \forall V1f \in (A_{27a}^{ty\_2Enum\_2Enum}).((p (ap (c_2Elist\_2ENULL \\
 & \quad A_{27a}) (ap (ap (c_2Elist\_2EGENLIST\ A_{27a})\ V1f)\ V0n))) \Leftrightarrow (V0n = c\_2Enum\_2E0)))) \\
 & \tag{89}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2EiZ (ap \\
& (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& ((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
(ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\
c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. \\
& (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Enum\_2ESUC V17n)))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
(ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& ((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{92}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{93}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{96}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
\end{aligned} \tag{99}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (100)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (101)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (102)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (103)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (104)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (105)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (106)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (107)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & \forall V0w \in (\text{ty\_2Efcp\_2Ecart } 2 A\_27a). (\forall V1i \in \text{ty\_2Enum\_2Enum}. \\ & ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2Efcp\_2Edimindex A\_27b) \\ & (c\_2Ebool\_2Ethel\_value A\_27b)))) \Rightarrow ((p (ap (ap (c\_2Efcp\_2Efcp\_index \\ & 2 A\_27b) (ap (c\_2Ewords\_2Ew2w A\_27a A\_27b) V0w)) V1i)) \Leftrightarrow ((p (ap \\ & (ap c\_2Eprim\_rec\_2E\_3C V1i) (ap (c\_2Efcp\_2Edimindex A\_27a) ( \\ & (c\_2Ebool\_2Ethel\_value A\_27a)))) \wedge (p (ap (ap (c\_2Efcp\_2Efcp\_index \\ & 2 A\_27a) V0w) V1i))))))) \end{aligned} \quad (108)$$

### Theorem 1

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0w \in (ty\_2Efcp\_2Ecart \\
& 2 A_{27a}). ((ap (c\_2Ewords\_2Ereduce\_xnor A_{27a}) V0w) = (ap (ap \\
& (c\_2Ebool\_2ELET (ty\_2Elist\_2Elist (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone)) \\
& (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone)) (\lambda V1l \in (ty\_2Elist\_2Elist \\
& (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone)). (ap (ap (ap (c\_2Elist\_2EFOLDL \\
& (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone) (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone)) \\
& (c\_2Ewords\_2Eword\_xnor ty\_2Eone\_2Eone)) (ap (c\_2Elist\_2EHD \\
& (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone)) V1l)) (ap (c\_2Elist\_2ETL \\
& (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone)) V1l)))) (ap (ap (c\_2Elist\_2EGENLIST \\
& (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone)) (\lambda V2i \in ty\_2Enum\_2Enum. \\
& (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum (ty\_2Efcpc\_2Ecart 2 ty\_2Eone\_2Eone)) \\
& (\lambda V3n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ewords\_2Eword\_extract \\
& A_{27a} ty\_2Eone\_2Eone) V3n) V3n) V0w))) (ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap (ap c\_2Earithmetic\_2E\_2D (ap (c\_2Efcpc\_2Edimindex A_{27a}) ( \\
& c\_2Ebool\_2Ethe\_value A_{27a}))) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V2i)))) \\
& (ap (c\_2Efcpc\_2Edimindex A_{27a}) (c\_2Ebool\_2Ethe\_value A_{27a})))))))
\end{aligned}$$