

thm_2Ewords_2E12w__w2l (TMR- mZWTkyt53bT9o1VGWduka6gg9Ctug1Br)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ x)) \text{ (} \lambda V1x \in 2.V1x \text{))} \text{ (} \lambda V1x \in 2.V1x \text{))}$

Definition 4 We define `c_2Ebool_2E_T` to be $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a})) \ (\lambda V1x \in 2.V1x)) \text{ (} \lambda V1x \in 2.V1x \text{))}$

Definition 6 We define `c_2Ebool_2E_F` to be $(\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2.V0t))$.

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2E_F) \ (\lambda V1t \in 2.V1t))$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2.V2t)) \text{ (} \lambda V2t \in 2.V2t \text{))}$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \tag{1}$$

Let `c_2Enum_2EREP_num` : ι be given. Assume the following.

$$c_2Enum_2EREP_num \in (\text{omega}^{ty_2Enum_2Enum}) \tag{2}$$

Let `c_2Enum_2ESUC_REP` : ι be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\text{omega}^{\text{omega}}) \tag{3}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\text{omega}}) \tag{4}$$

Definition 10 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num$

Definition 11 We define $c_Eprim_rec_E3C$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 12 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 13 We define $c_Earithmetic_EMIN$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Let $c_Enum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Enum_EZERO_REP \in \omega \tag{5}$$

Definition 14 We define c_Enum_E0 to be $(ap\ c_Enum_EABS_num\ c_Enum_EZERO_REP)$.

Definition 15 We define $c_Earithmetic_EZERO$ to be c_Enum_E0 .

Let $c_Earithmetic_E2B : \iota$ be given. Assume the following.

$$c_Earithmetic_E2B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \tag{6}$$

Definition 16 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmetic$

Definition 17 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \tag{7}$$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \tag{8}$$

Definition 18 We define $c_Ebit_EDIV_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Let $c_Earithmetic_E2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E2D \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \tag{9}$$

Let $c_Earithmetic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmetic_EMOD \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \tag{10}$$

Definition 19 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 20 We define c_Ebit_EBITS to be $\lambda V0h \in ty_Enum_Enum.\lambda V1l \in ty_Enum_Enum.\lambda V$

Definition 21 We define $c_Ecombin_ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 22 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1$

Definition 23 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 24 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (12)$$

Definition 25 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 26 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 27 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 28 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap c_2Enum_2ESUC (ap$

Definition 29 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 30 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 31 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2E$

Let $c_2Enumeral_2Eexp_help : \iota$ be given. Assume the following.

$$c_2Enumeral_2Eexp_help \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Definition 32 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic$

Let $ty_2EfcP_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2EfcP_2Efinite_image A0) \quad (15)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (16)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (17)$$

Let $c_2EfcP_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2EfcP_2Edimindex A_27a \in (ty_2Enum_2Enum)^{(ty_2Ebool_2Eitself A_27a)} \quad (18)$$

Definition 33 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap c_2Ebool_2E_2F_5C$

Definition 34 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota. (ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Efcp_2Ecart A0 A1) \quad (19)$$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Efcp_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \quad (20)$$

Definition 35 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efcp_2Ecart A_27a$

Definition 36 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2. \lambda V1n \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 37 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap$

Definition 38 We define c_2Efcp_2EFCP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0g \in (A_27a^{ty_2Enum_2Enum}). (ap$

Definition 39 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efcp_2Ecart 2 A_27a). (ap (ap$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (22)$$

Let $c_2Enumposrep_2En2l : \iota$ be given. Assume the following.

$$c_2Enumposrep_2En2l \in (((ty_2Elist_2Elist ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (23)$$

Definition 40 We define $c_2Ewords_2Ew2l$ to be $\lambda A_27a : \iota. \lambda V0b \in ty_2Enum_2Enum. \lambda V1w \in (ty_2Efcp$

Let $c_2Enumposrep_2El2n : \iota$ be given. Assume the following.

$$c_2Enumposrep_2El2n \in ((ty_2Enum_2Enum^{(ty_2Elist_2Elist ty_2Enum_2Enum)})^{ty_2Enum_2Enum}) \quad (24)$$

Definition 41 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota. \lambda V0n \in ty_2Enum_2Enum. (ap (c_2Efcp_2EFCP$

Definition 42 We define $c_2Ewords_2El2w$ to be $\lambda A_27a : \iota. \lambda V0b \in ty_2Enum_2Enum. \lambda V1l \in (ty_2Elist$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP \\
& V0m) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2n \in \\
& ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP V1m) (ap c_2Enum_2ESUC \\
& V2n)) = (ap (ap c_2Earithmetic_2E_2A V1m) (ap (ap c_2Earithmetic_2EEXP \\
& V1m) V2n))))))
\end{aligned} \tag{25}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)) \tag{26}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge ((ap (\\
& ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V1n)))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\
& V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))
\end{aligned} \tag{27}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
(ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
V1n) V0m)))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
(ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\
V1n) V0m)))) \tag{29}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
& (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))))
\end{aligned} \tag{30}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
(ap c_2Enum_2ESUC V0m)) V1n)))))) \tag{31}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (32)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0)) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) V0m) = V0m)) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m)) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) V0m) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))))))) \quad (38)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V1n) V0m)))) \quad (39)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A V1n) V0m)))) \quad (40)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A V1n) V2p))))))) \quad (41)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V0m) (ap (ap c_2Earithmetic_2E_2A V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) V2p)))))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = V0m) \Leftrightarrow (V1n = c_2Enum_2E0)))) \quad (43)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \quad (44)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V0m))) \quad (45)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum.(V0m = (ap (ap c_2Earithmetic_2E_2B V1n) (ap (ap c_2Earithmetic_2E_2B V2p) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))))))) \quad (46)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Enum_2Enum. (\forall V1q \in ty_2Enum_2Enum. (\\
& \quad \forall V2n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP V2n) \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0p) V1q)) = (ap (ap c_2Earithmetic_2E_2A \\
& \quad (ap (ap c_2Earithmetic_2EEXP V2n) V0p)) (ap (ap c_2Earithmetic_2EEXP \\
& \quad \quad V2n) V1q))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\
& \quad \quad (V1n = (ap (ap c_2Earithmetic_2E_2B V0m) V2p))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2E_2A V0m) V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = \\
& \quad \quad c_2Enum_2E0) \vee (V1n = c_2Enum_2E0))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& \quad \quad ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& \quad \quad V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad \quad (ap c_2Enum_2ESUC V1n)) V0m))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& \quad c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad \quad c_2Earithmetic_2EZERO))) V0n)))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& \quad ((ap (ap c_2Earithmetic_2EEXP V0n) V1m) = c_2Enum_2E0) \Leftrightarrow ((V0n = \\
& \quad \quad c_2Enum_2E0) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1m))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& V0n) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) \wedge ((ap (ap c_2Earithmetic_2EEXP V0n) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& V0n)))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& V0b)) \Rightarrow (\forall V1n \in ty_2Enum_2Enum.(\forall V2m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2EEXP \\
& V0b) V2m)) (ap (ap c_2Earithmetic_2EEXP V0b) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V2m) V1n))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b1 \in ty_2Enum_2Enum.(\forall V1b2 \in ty_2Enum_2Enum. \\
& (\forall V2x \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EEXP \\
& V0b1) V2x) = (ap (ap c_2Earithmetic_2EEXP V1b2) V2x)) \Leftrightarrow ((V2x = c_2Enum_2E0) \vee \\
& (V0b1 = V1b2))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(((ap (ap c_2Earithmetic_2EMIN V0n) \\
& V0n) = V0n))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum.(\forall V1l \in ty_2Enum_2Enum.(\\
& (ap (ap (ap c_2Ebit_2EBITS V0h) V1l) c_2Enum_2E0) = c_2Enum_2E0)))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum.(\neg (p (ap (ap c_2Ebit_2EBIT V0b) c_2Enum_2E0))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum.(p (ap (ap c_2Ebit_2EBIT V0b) (ap \\
& (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO))) V0b))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Enum_2Enum. (\forall V1b \in ty_2Enum_2Enum. (\\
& (\neg(V0a = V1b)) \Rightarrow (\neg(p (ap (ap c_2Ebit_2EBIT V0a) (ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& V1b))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h1 \in ty_2Enum_2Enum. (\forall V1l1 \in ty_2Enum_2Enum. \\
& (\forall V2h2 \in ty_2Enum_2Enum. (\forall V3l2 \in ty_2Enum_2Enum. \\
& (\forall V4n \in ty_2Enum_2Enum. ((ap (ap (ap c_2Ebit_2EBITS V2h2) \\
V3l2) (ap (ap (ap c_2Ebit_2EBITS V0h1) V1l1) V4n)) = (ap (ap (ap c_2Ebit_2EBITS \\
& (ap (ap c_2Earithmetic_2EMIN V0h1) (ap (ap c_2Earithmetic_2E_2B \\
V2h2) V1l1))) (ap (ap c_2Earithmetic_2E_2B V3l2) V1l1)) V4n))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum. (\forall V1l \in ty_2Enum_2Enum. (\\
& \forall V2a \in ty_2Enum_2Enum. (\forall V3b \in ty_2Enum_2Enum. ((\\
& p (ap (ap c_2Eprim_rec_2E_3C V3b) (ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
V1l))) \Rightarrow ((ap (ap (ap c_2Ebit_2EBITS V0h) V1l) (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V2a) (ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
V1l))) V3b) = (ap (ap (ap c_2Ebit_2EBITS V0h) V1l) (ap (ap c_2Earithmetic_2E_2A \\
V2a) (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) V1l))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0h \in ty_2Enum_2Enum. (\forall V1l \in ty_2Enum_2Enum. (\\
& \forall V2a \in ty_2Enum_2Enum. (\forall V3b \in ty_2Enum_2Enum. ((\\
& ap (ap (ap c_2Ebit_2EBITS V0h) V1l) (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V2a) (ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& (ap c_2Enum_2ESUC V0h)))) V3b) = (ap (ap (ap c_2Ebit_2EBITS V0h) \\
V1l) V3b))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1h \in ty_2Enum_2Enum. (\\
& \forall V2l \in ty_2Enum_2Enum. (\forall V3a \in ty_2Enum_2Enum. ((\\
& p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B \\
V2l) V0n)) V1h) \Rightarrow ((p (ap (ap c_2Ebit_2EBIT V0n) (ap (ap (ap c_2Ebit_2EBITS \\
V1h) V2l) V3a))) \Leftrightarrow (p (ap (ap c_2Ebit_2EBIT (ap (ap c_2Earithmetic_2E_2B \\
V2l) V0n)) V3a))))))
\end{aligned} \tag{66}$$

Assume the following.

$$True \quad (67)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (68)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (69)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (70)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (71)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \wedge ((p \ V1t2) \wedge (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \wedge (p \ V2t3)))))) \quad (72)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (73)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (74)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (75)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (76)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (77)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (78)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (79)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (80)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (ap\ V1Q\ V4x)))))))))) \quad (81)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (82)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (83)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (84)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \vee (p\ V1B)))))) \quad (85)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (86)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (87)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2)))))) \quad (88)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))) \quad (89)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}. (\forall V3x_{.27} \in A_{.27a}. (\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_2ECOND } A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_2ECOND } A_{.27a}) V1Q) V3x_{.27} \\ & V5y_{.27})))))) \quad (90) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}. (\forall V1t2 \in \\ & A_{.27a}. ((ap (ap (ap (c_{.2Ebool_2ECOND } A_{.27a}) c_{.2Ebool_2ET} V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}. (\forall V3t2 \in A_{.27a}. ((ap \\ & (ap (ap (c_{.2Ebool_2ECOND } A_{.27a}) c_{.2Ebool_2EF} V2t1) V3t2) = V3t2)))) \quad (91) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0f \in ((A_{.27a}^{A_{.27a}})^{A_{.27a}}). \\ & ((\forall V1x \in A_{.27a}. (\forall V2y \in A_{.27a}. (\forall V3z \in A_{.27a}. \\ & ((ap (ap V0f V1x) (ap (ap V0f V2y) V3z)) = (ap (ap V0f (ap (ap V0f V1x) \\ & V2y)) V3z)))) \Rightarrow ((\forall V4x \in A_{.27a}. (\forall V5y \in A_{.27a}. ((ap \\ & (ap V0f V4x) V5y) = (ap (ap V0f V5y) V4x)))) \Rightarrow (\forall V6x \in A_{.27a}. (\\ & \forall V7y \in A_{.27a}. (\forall V8z \in A_{.27a}. ((ap (ap V0f V6x) (ap (ap \\ & V0f V7y) V8z)) = (ap (ap V0f V7y) (ap (ap V0f V6x) V8z)))))) \quad (92) \end{aligned}$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}. ((ap (c_{.2Ecombin_2EI} A_{.27a}) V0x) = V0x)) \quad (93)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in (ty_2EfcP_2Ecart\ A_27a\ A_27b). (\forall V1y \in (ty_2EfcP_2Ecart \\
& A_27a\ A_27b). ((V0x = V1y) \Leftrightarrow (\forall V2i \in ty_2Enum_2Enum. ((p\ (ap \\
& (ap\ c_2Eprim_rec_2E_3C\ V2i)\ (ap\ (c_2EfcP_2Edimindex\ A_27b)\ (\\
& c_2Ebool_2Ethe_value\ A_27b)))))) \Rightarrow ((ap\ (ap\ (c_2EfcP_2EfcP_index \\
& A_27a\ A_27b)\ V0x)\ V2i) = (ap\ (ap\ (c_2EfcP_2EfcP_index\ A_27a\ A_27b) \\
& V1y)\ V2i))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0g \in (A_27a^{ty_2Enum_2Enum}). (\forall V1i \in ty_2Enum_2Enum. \\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1i)\ (ap\ (c_2EfcP_2Edimindex\ A_27b) \\
& (c_2Ebool_2Ethe_value\ A_27b)))))) \Rightarrow ((ap\ (ap\ (c_2EfcP_2EfcP_index \\
& A_27a\ A_27b)\ (ap\ (c_2EfcP_2EFCP\ A_27a\ A_27b)\ V0g))\ V1i) = (ap\ V0g \\
& V1i))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{99}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO) \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))))))))) \\
& \tag{100}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ c_2Earithmic_2EZERO)\ V0n)) \Leftrightarrow \\
& True) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow (\neg(p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V1m)\ V0n)))) \wedge (((p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ c_2Earithmic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D \\
& V0n)\ V1m))))))))) \\
& \tag{101}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) \wedge (((ap (ap c_2Earithmetic_2EEXP (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V0n))) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Enumeral_2Etex_help \\
& (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 V0n))) c_2Earithmetic_2EZERO))) \wedge \\
& ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n))) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap (ap c_2Enumeral_2Etex_help (ap c_2Earithmetic_2EBIT1 V0n)) \\
& c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0b)) \Rightarrow ((\\
& ap (ap c_2Enumposrep_2El2n V0b) (ap (ap c_2Enumposrep_2En2l V0b) \\
& V1n)) = V1n))))
\end{aligned} \tag{103}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg (p (ap (ap c_2Eprim_rec_2E_3C V0n) c_2Enum_2E0)))) \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V0m) (ap c_2Enum_2ESUC V1n))) \Leftrightarrow (\\
& (V0m = V1n) \vee (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))))))
\end{aligned} \tag{105}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{106}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{108}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (109)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (110)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (111)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))) \quad (112)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (113)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (114)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (115)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (116)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (117)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (118)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (119)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (120)$$

Assume the following.

$$\begin{aligned} & ((\forall V0f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}).((ap (ap c_2Esum_num_2ESUM \\ & \quad c_2Enum_2E0) V0f) = c_2Enum_2E0)) \wedge (\forall V1m \in ty_2Enum_2Enum. \\ & (\forall V2f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}).((ap (ap c_2Esum_num_2ESUM \\ & \quad (ap c_2Enum_2ESUC V1m)) V2f) = (ap (ap c_2Earithmetic_2E_2B (ap \\ & \quad (ap c_2Esum_num_2ESUM V1m) V2f)) (ap V2f V1m))))))) \end{aligned} \quad (121)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0b \in ty_2Enum_2Enum.(\\ & \quad \forall V1w \in (ty_2EfcP_2Ecart\ 2\ A_27a).((p (ap (ap c_2Eprim_rec_2E_3C \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\ & V0b)) \Rightarrow ((ap (ap (c_2Ewords_2E!2w\ A_27a) V0b) (ap (ap (c_2Ewords_2Ew2! \\ & \quad A_27a) V0b) V1w)) = V1w)))) \end{aligned}$$