

thm_2Ewords_2En2w__itself__ind
(TMZb8yQhYnd17c36ujePkzyL4dhzf2hFQQQT)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty (ty_2Ebool_2Eitself\ A0) \tag{2}$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \tag{3}$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty (ty_2Epair_2Eprod\ A0\ A1) \tag{4}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \tag{5}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 9 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Erelation_2EEMPTY_REL$ to be $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27a.c_2E$

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Definition 12 We define $c_2Erelation_2EWF$ to be $\lambda A.27a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).(ap (c_2Ebool_2E_21$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{7}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg (p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Ebool_2Eitself A.27a)}). \\ & ((p (ap V0P (c_2Ebool_2Ethe_value A.27a))) \Rightarrow (\forall V1i \in (ty_2Ebool_2Eitself \\ & A.27a).(p (ap V0P V1i)))))) \end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod A.27a A.27b).(\exists V1q \in A.27a. \\ & (\exists V2r \in A.27b.(V0x = (ap (ap (c_2Epair_2E_2C A.27a A.27b) \\ & V1q) V2r)))))) \end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ & ((p\ (ap\ (c_2Erelation_2EWF\ A_27a)\ V0R)) \Rightarrow (\forall V1P \in (2^{A_27a}). \\ & ((\forall V2x \in A_27a. ((\forall V3y \in A_27a. ((p\ (ap\ (ap\ V0R\ V3y)\ V2x)) \Rightarrow \\ & (p\ (ap\ V1P\ V3y)))) \Rightarrow (p\ (ap\ V1P\ V2x)))) \Rightarrow (\forall V4x \in A_27a. (p\ (ap \\ & V1P\ V4x)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (c_2Erelation_2EWF\ A_27a)\ (c_2Erelation_2EEMPTY_REL\ A_27a))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (16)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (20)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))) \quad (21)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Ebool_2Eitself\ A_27a))}). \\ & ((\forall V1n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C \\ & ty_2Enum_2Enum\ (ty_2Ebool_2Eitself\ A_27a))\ V1n)\ (c_2Ebool_2Ethe_value \\ & A_27a)))) \Rightarrow (\forall V2v \in ty_2Enum_2Enum. (\forall V3v1 \in (ty_2Ebool_2Eitself \\ & A_27a). (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum\ (ty_2Ebool_2Eitself \\ & A_27a))\ V2v)\ V3v1)))))) \end{aligned}$$