

thm\_2Ewords\_2Esw2sw\_0  
 (TMSy4Q7HiRTLcKmvQvWWiBJFj4gZKBLzpvv)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (6)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c_2$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT1 n) V0)$

**Definition 8** We define  $\text{c\_2Earthmetic\_2ENUMERAL}$  to be  $\lambda V0x \in \text{ty\_2Enum\_2Enum}.V0x.$

**Definition 9** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT2 n) V0)$

Let  $c_2$  be given. Assume the following.

Let  $c_2Ea$  in metric  $\mathbb{Z}^d\Lambda T$ . It be given. Assume the following.

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

Let  $c_2Earthmetc_2EDIV : t$  be given. Assume the following.

$$c\_2Earithmetic\_2EDI \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 10** We define `c_2EBit_2EDIV_2EXP` to be  $\lambda V.0x \in ty\_2Enum\_2Enum. \lambda V.1n \in ty\_2Enum\_2Enum$

Let  $c_2Earithmetic_2E_2D : \iota$  be given. Assume the following.

$$c\_2Earthmetic\_2E\_2D \in ((ty\_2Enum\_2Enum_ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{(10)}$$

**Definition 11** We define  $c_2EBit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 12** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum . \lambda V1l \in ty\_2Enum\_2Enum . \lambda V2m \in ty\_2Enum\_2Enum . \lambda V3n \in ty\_2Enum\_2Enum . \lambda V4o \in ty\_2Enum\_2Enum . \lambda V5p \in ty\_2Enum\_2Enum . \lambda V6q \in ty\_2Enum\_2Enum . \lambda V7r \in ty\_2Enum\_2Enum . \lambda V8s \in ty\_2Enum\_2Enum . \lambda V9t \in ty\_2Enum\_2Enum . \lambda V10u \in ty\_2Enum\_2Enum . \lambda V11v \in ty\_2Enum\_2Enum . \lambda V12w \in ty\_2Enum\_2Enum . \lambda V13x \in ty\_2Enum\_2Enum . \lambda V14y \in ty\_2Enum\_2Enum . \lambda V15z \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 14** We define  $c_{\text{Ebool}} \cdot \text{ELET}$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0f \in (A.27b^{A \rightarrow 27a}).(\lambda V1x \in A.27$

**Definition 15** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$

**Definition 16** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_5C\_2F\ t1)\ t2)))$

**Definition 17** We define  $\varsigma \in \text{2Ebool\_2EE}$  to be (ap,  $(\varsigma \in \text{2Ebool\_2E 21 2})$ ,  $(\lambda V0t \in 2.V0t))$ .

**Definition 18** We define  $\in_2$  to be  $\lambda V. 0t_1 \in_2 (\lambda V. 1t_2 \in_2 (\dots))$ , where  $\in_2$  is defined as above.

**Definition 19.** We define  $c \in 2\text{Emin}(\mathcal{A}, \mathcal{P})$  to be  $\lambda A. \lambda P \in 2^A$  if  $(\exists x \in A) p_c(x)$ , then  $(\lambda x. x \in A) \wedge$

**Definition 29** We define a 2EVAL-2SECOND to be a 4.27- $\epsilon$ - $(\lambda V)t \in 2$ ,  $(\lambda V1t1 \in 4.27)$ ,  $(\lambda V2t2 \in 4.27)$ .

**Definition 22** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27b}))$

**Definition 23** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a) (A\_27a^{A\_27a})) A\_27a)$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 24** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E))$

**Definition 25** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 26** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 27** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 28** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 29** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 30** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 31** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (14)$$

Let  $c\_2Ebool\_2Ethethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ebool\_2Ethethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (15)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Efcp\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (16)$$

**Definition 32** We define  $c\_2Ebit\_2ESIGN\_EXTEND$  to be  $\lambda V0l \in ty\_2Enum\_2Enum. \lambda V1h \in ty\_2Enum\_2Enum$

Let  $ty\_2Efcp\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Efcp\_2Efinite\_image A0) \quad (17)$$

**Definition 33** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C))$

**Definition 34** We define  $c\_2Efcp\_2Efinite\_index$  to be  $\lambda A\_27a : \iota. (ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum}))$

Let  $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty A0 \Rightarrow & \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcp\_2Ecart \\ & A0 A1) \end{aligned} \quad (18)$$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow & \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart \\ & A\_27a A\_27b \in ((A\_27a^{ty\_2Efcp\_2Efinite\_image A\_27b}) (ty\_2Efcp\_2Ecart A\_27a A\_27b)) \end{aligned} \quad (19)$$

**Definition 35** We define  $c\_2Efcp\_2Efcp\_index$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a A\_27b)$

**Definition 36** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2. \lambda V1n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (20)$$

**Definition 37** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27a). (ap (ap (ap$

**Definition 38** We define  $c\_2Efcp\_2EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}). (ap (ap (ap (c\_2Ebool$

**Definition 39** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_2Enum\_2Enum. (ap (c\_2Efcp\_2EFC$

**Definition 40** We define  $c\_2Ewords\_2Esw2sw$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w \in (ty\_2Efcp\_2Ecart 2 A\_27b)$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \quad (21) \\ c\_2Enum\_2E0) = V0m))$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1k \in ty\_2Enum\_2Enum. ( \\ (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1k) V0n)) \Rightarrow ((ap (ap c\_2Earithmetic\_2EMOD \\ V1k) V0n) = V1k)))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty\_2Enum\_2Enum. (\forall V1y \in ty\_2Enum\_2Enum. ( \\ (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap (ap c\_2Earithmetic\_2EXP \\ V0x) V1y))) \Leftrightarrow ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) V0x)) \vee \\ (V1y = c\_2Enum\_2E0)))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0b \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Ebit\_2EBIT V0b) c\_2Enum\_2E0)))) \quad (24)$$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}). (\forall V1x \in A\_27a. ((ap (ap (c\_2Ebool\_2ELET \\ & A\_27a A\_27b) V0f) V1x) = (ap V0f V1x))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))) \end{aligned} \quad (29)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. ((ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))) \quad (33)$$

Assume the following.

$$\begin{aligned}
 & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
 & (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\
 & (\forall V5y\_27 \in A\_27a.(((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\
 & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\
 & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\
 & V5y\_27)))))))
 \end{aligned} \tag{34}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Ecombin\_2EI\ \\
 A\_27a)\ V0x) = V0x)) \tag{35}$$

Assume the following.

$(\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B c_2Enum\_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B V1n) c_2Enum\_2E0) = V1n)) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B V3m) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Enum\_2EiZ (ap (ap c_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge (\forall V4n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A c_2Enum\_2E0) V4n) = c_2Enum\_2E0)) \wedge (\forall V5n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A V5n) c_2Enum\_2E0) = c_2Enum\_2E0)) \wedge (\forall V6n \in ty\_2Enum\_2Enum. (\forall V7m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A (ap c_2Earithmetic\_2ENUMERAL V6n)) (ap c_2Earithmetic\_2ENUMERAL V7m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2E\_2A V6n) V7m)))))) \wedge (\forall V8n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D c_2Enum\_2E0) V8n) = c_2Enum\_2E0)) \wedge (\forall V9n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D V9n) c_2Enum\_2E0) = V9n)) \wedge (\forall V10n \in ty\_2Enum\_2Enum. (\forall V11m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D (ap c_2Earithmetic\_2ENUMERAL V10n)) (ap c_2Earithmetic\_2ENUMERAL V11m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2E\_2D V10n) V11m)))))) \wedge (\forall V12n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 V12n)))) = c_2Enum\_2E0)) \wedge (\forall V13n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT2 V13n)))) = c_2Enum\_2E0)) \wedge (\forall V14n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP V14n) c_2Enum\_2E0) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))))) \wedge (\forall V15n \in ty\_2Enum\_2Enum. (\forall V16m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP (ap c_2Earithmetic\_2ENUMERAL V15n)) (ap c_2Earithmetic\_2ENUMERAL V16m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2EEEXP V15n) V16m)))))) \wedge (((ap c_2Enum\_2ESUC c_2Enum\_2E0) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))) \wedge (\forall V17n \in ty\_2Enum\_2Enum. ((ap c_2Enum\_2ESUC (ap c_2Earithmetic\_2ENUMERAL V17n)) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Enum\_2ESUC V17n)))))) \wedge (((ap c_2Eprim\_rec\_2EPRE c_2Enum\_2E0) = c_2Enum\_2E0) \wedge (\forall V18n \in ty\_2Enum\_2Enum. ((ap c_2Eprim\_rec\_2EPRE (ap c_2Earithmetic\_2ENUMERAL V18n)) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Eprim\_rec\_2EPRE V18n)))))) \wedge (\forall V19n \in ty\_2Enum\_2Enum. (((ap c_2Earithmetic\_2ENUMERAL V19n) = c_2Enum\_2E0) \Leftrightarrow (V19n = c_2Earithmetic\_2EZERO))) \wedge (\forall V20n \in ty\_2Enum\_2Enum. ((c_2Enum\_2E0 = (ap c_2Earithmetic\_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic\_2EZERO))) \wedge (\forall V21n \in ty\_2Enum\_2Enum. ((\forall V22m \in ty\_2Enum\_2Enum. (((ap c_2Earithmetic\_2ENUMERAL V21n) = (ap c_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge ((\forall V23n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C V23n) c_2Enum\_2E0)) \Leftrightarrow False))) \wedge (\forall V24n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2EZERO) V24n)))) \wedge (\forall V25n \in ty\_2Enum\_2Enum. (\forall V26m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2ENUMERAL V25n) (ap c_2Earithmetic\_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V27n)) \Leftrightarrow False))) \wedge (\forall V28n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL V28n)) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2EZERO) V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V29n)) \Leftrightarrow True))) \wedge (\forall V30m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V30m)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V31n)) \Leftrightarrow True))) \wedge (\forall V32n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) (ap c_2Earithmetic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& (ap c_2Earithmetic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT1 V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg(p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2EBIT2 V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))) \\
\end{aligned} \tag{37}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((ap (c_2Ewords_2Ew2n A\_27a) (ap \\
(c_2Ewords_2En2w A\_27a) c_2Enum_2E0)) = c_2Enum_2E0) \tag{38}$$

### Theorem 1

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\
& (ap (c_2Ewords_2Esw2sw A\_27a A\_27b) (ap (c_2Ewords_2En2w A\_27a) \\
& c_2Enum_2E0)) = (ap (c_2Ewords_2En2w A\_27b) c_2Enum_2E0))
\end{aligned}$$